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The Brown Boveri Computer Centre

The analogue computer

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CONTENTS:

THE BROWN BOVERI COMPUTER CENTRE

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The present article describes why Brown Boveri found it necessary to install computers, intended solely for technical and scientific services, and deals with the organization of the Centre in which they are housed. In addition to the a.c. network analyser there is an analogue and a digital computer. The article concludes with an account of some of the problems of organization associated with the operation of the digital computer.

The Need

IT belong to the characteristic trends of modern technical development that the need of the engineer for means of simplifying calculation work should have grown at an almost unbelievable rate. There are various reasons for this phenomenon. One of the most important is the enormous increase in the unit capacities of the machines which have to be designed. In order to make the best use of the materials on the one hand, while avoiding the risk of difficulties later when the machine is in service, it is necessary to determine certain properties of the machine—for instance the forces produced by a short circuit, or the temperature rises in the copper and iron—much more exactly than for a smaller machine. Moreover, in many cases the Company is obliged to give quite extensive guarantees, for example regarding the behaviour of a regulator in the event of a particular kind of disturbance. The ability to uphold such guarantees is often dependent on the results of preliminary calculation or experiments.

Apart from the experiments during development, which continue to retain their full significance, the aids for calculation become increasingly important. The necessity for their employment primarily arises from the efforts made to improve the quality of technical products. Of course cases may well be visualized, particularly during the calculation work for new designs, where the ability to undertake the complete calculation of several variants allows the manufacturing costs to be reduced. But, from the experience gained so far, this factor alone does not justify the purchase of a large computer, from the economic aspect. The development referred to above is taken into account by Brown Boveri. Since December 1959 work has been in progress in a true Computer Centre.

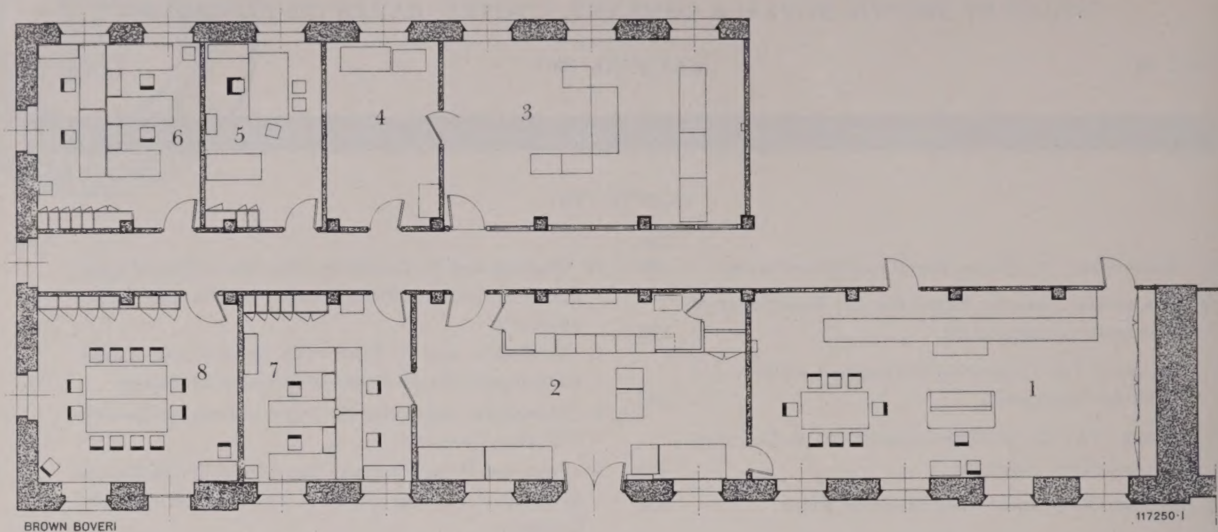

Fig. 1. – Plan view of the Brown Boveri Computer Centre

1 = Network analyser room
2 = Analogue computer room
3 = Digital computer room
4 = Preparation room
5–7 = Offices
8 = Conference room

Fig. 2. – Analogue computer

The various components are housed in the cabinets on the left. The switchboard on which the program for the particular calculation is set up can be clearly seen. To its right are an oscillograph and, behind it, a type KC voltage regulator.

This possesses three of the most important "calculating machines", namely an a.c. network analyser, an analogue computer and, as most recent but at the same time the largest member of the family, a digital computer which may be regarded as a calculating machine in the true sense of the word. The a.c. network analyser, which has rendered invaluable service for many years, is in effect a model of the network on which actual phenomena can be simulated. The analogue computer is also more of a model than a true computer. Admittedly the relationship between image and reality is less close in this case.

Of course there are other mathematical aids in the factory, outside the Computer Centre, which may also be regarded as calculating machines, such as the electrolytic tank and the analogue model used for investigating the critical speeds of high-speed shafts [1, 2].

Layout of the Accommodation

Since it was commissioned in 1953, the a.c. network analyser has been housed in the cellar of the main administrative building of the Company. It was therefore an obvious solution to locate the analogue and digital computers in adjacent rooms, there-

Fig. 3. – A.C. network analyser

Panel with active and passive elements and switchboard. On the left is the metering desk with automatic devices for calling up the elements, and light-spot instruments.

by creating a Computer Centre. Fig. 1 shows the plan view of the Centre. The analogue computer and the network analyser (Fig. 2 and 3, respectively) are housed in adjoining rooms. With this arrangement it is possible to establish a connection between the two, as it is to be expected that problems will occur, demanding the combined efforts of the two machines. The digital computer (Fig. 4) also has a room of its own, in which the operator in charge of the machine can work undisturbed; but often the person concerned with the calculation being handled is also present. The punched tapes for programs and other data are prepared in the adjoining preparation room.

Naturally the operation of the Computer Centre needs a sufficiently large staff of experts, for whom offices are provided close at hand. Since the Centre has to serve a wide circle of "customers", a conference room was also included. In it small groups can hold discussions, but it can also be used for internal lectures or courses of instruction. All rooms are air-conditioned, so that good and uniform working conditions are provided for the electronic computers.

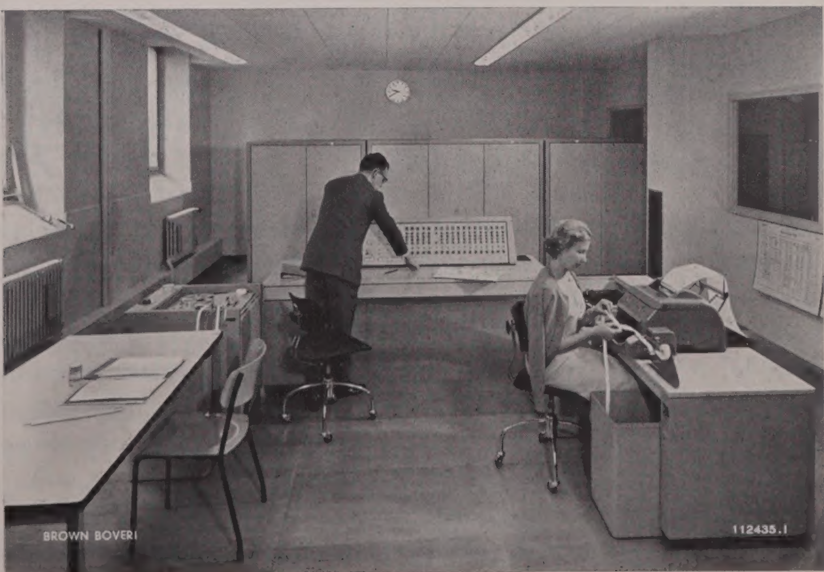

Fig. 4. – Digital computer

The computing and control sections with the magnetic-core memory and magnetic drum store are in the cabinets in the rear. On the left the punched-tape input equipment; in the middle is the operator's desk with the output equipment (punching machine and page printer) on the right.

Technical Data of the Digital Computer

Reports on the analogue computer and network analyser have appeared in earlier numbers of this Review [3,4], so that further reference to their details is superfluous. On the other hand, the digital computer (Siemens & Halske type 2002) deserves special mention. By modern standards this is a medium-sized machine. The model installed in the Company's premises is ideal for technical and scientific duties. This means, among other things, that the computer itself, to judge by other electronic equipment, operates very rapidly, while the input and output sections are not quite so luxurious. Furthermore, for the applications involved it is important for numbers processed by the floating-point system not to be handled any slower or with any additional complications than they would by the fixed-point system, which is commonly used for commercial applications.

Punched tapes are used for the input and output of information, facilities also being provided for the results of a calculation to be printed out in clear by a page printer. Advantage is seldom taken of this facility owing to the relatively slow printing speed of the page printer. For storing the program and data there is a magnetic-core memory capable of holding 2000 words and a magnetic drum for 10 000 words.

As regards the working speed of the computer it is not easy to make any binding statement, since this varies greatly from one problem to another. On the average, about 2000 operations (arithmetical and logical operations) are performed per second.

Questions of Organization

In conclusion it is worth saying something about the questions arising with respect to the organization of a Computing Centre of this nature to serve a large industrial undertaking. The field of application of a digital computer in the technical and scientific sphere in a company having the structure of Brown Boveri is extraordinarily broad, as may be gathered from the different subjects dealt with in the articles in this issue. Not only typical mathematical problems are dealt with (e.g. the solution of differential equations); equally important are tasks which may not involve so much mathematical ability, but where series of routine calculations formerly carried out laboriously by hand can be handed over to the digital computer.

There are two fundamental ways of running a computer centre of this kind; one is the "closed shop", the other the "open shop". When the former principle is adopted, the problems to be handled by the computer are given to the staff of the Computer Centre in a concretely formulated form by the engineering departments. The Centre staff then carry out all tasks involved, until a numerical result is obtained. These tasks are: the determination of the mathematical method, the preparation of the block and flow diagram, encoding the instructions, checking the program and, finally, performing the calculation according to the tested program, using the input data provided.

In contrast, with the "open shop" system the greater part of the programming work is undertaken by the particular engineering department concerned. The Computer Centre then provides a technical service, acting in more of an advisory capacity; it enables the various department to exchange their experience, is responsible for the operation of the computer and—as one of its most important duties—ensures that its library of generally applicable subroutines is utilized to the best advantage.

Both systems naturally have their advantages and disadvantages. With the "open shop" system it is much quicker to spread the knowledge regarding the possible methods of electronic data processing in the engineering departments. Moreover, experience has shown that, for many tasks, the goal is attained much more quickly if an engineer of the specialist department is given instruction in the art of programming, than when a trained programmer has to delve into the special problems associated with a particular task. These advantages are offset by the undeniable drawback that the programs produced by the open shop method are, on the average, less polished than those obtainable with skilled programmers operating as a closed shop. Weighing up the pros and cons it was decided from the very start to aim at operating the Computer Centre as an open shop. Several months before the computer was installed, internal program-

ming courses were held, so that now every engineering department has at least one trained programmer. Experience gained in the meantime strengthens our conviction that this approach was right. Shortly after the "running-in period" had expired the computer was fully occupied; but this, of course, is no proof that the time spent in calculation was utilized to the best advantage. In the early stages allowance must be made for time being lost in checking the programs, owing to the lack of experience of the programmers. These are typical teething troubles, though, which can soon be overcome by well directed and rigorously executed measures. One of these, for instance, is to insist on the computer being handled only by the operator and not by the members of the engineering departments, even when checking through the program. Finally, experience has shown that a suitable form of organization can always be found, even for special cases, provided development is followed with care and suitable precautions are taken. In the Brown Boveri Computer Centre the organization is nearer the "open shop" than the "closed shop", although it exhibits certain features of the latter. This applies above all to scientific problems necessitating a very large amount of mathematical work. In such cases the Centre acts rather like a research department.

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ON THE IMPORTANCE OF MATHEMATICS TO ENGINEERING

51:62

With the aid of some typical examples, the present article discusses the mutual relationships between engineering and mathematics. Following a review of the pioneering days in electrical engineering, an attempt is made to find a means of determining the present state of the art.

THE MATHEMATICIAN is, or at least until quite recently was, looked upon as an unworldly idealist, paying little heed to the possible applications for his scientific knowledge. This widespread impression is based on false assumptions though; for it is the great mathematicians in history who have repeatedly striven to place their ability at the service of physics or engineering. To mention but a few great names, readers are reminded of such men as Newton, Euler and Gauss. It is well known that Newton and Leibnitz were almost coincident in the creation of the infinitesimal calculus. By adopting new definitions Newton succeeded in formulating his famous laws of motion, which today are still the recognized foundation of the classical mechanics. Euler dealt with quite a number of technical problems; he, for instance, is the father of the basic turbine equations. Finally Gauss—"a prince among mathematicians"—among his many achievements, rendered an invaluable contribution to the theory of electromagnetism.

The number of mathematicians employed in industrial untertakings nowadays would have been beyond all imagination ten or twenty years ago. At the same time too, many engineers have experienced the need for expressing more and more technical relationships by mathematical means, with the accompanying trend towards a mathematical way of thinking. The fact that advantage can only be taken of the facilities offered by modern computers when there is a staff of skilled mathematicians available to develop expedient numerical methods, is only a partial explanation for the above trend. We, rather, are more inclined to regard this as an expression of the fact that modern engineering exhibits a pronounced trend towards the abstract. It is therefore most attractive to endeavour to trace this phenomenon and, after a brief review of past history, to try to determine the present state of the art. In doing so, remarks will be principally confined to the field of heavy electrical engineering.

Mathematical Thought in the Everyday Life of the Engineer

When, in the remarks that follow, reference is made to mathematicians and engineers, these two professions must be understood as types; it may well be that, in reality, an engineer possesses many of the qualities of a mathematician, and vice versa. But an engineer and a mathematician think on entirely different lines from one another. Those of the mathematician are centred on logic and the abstract, whereas the primary aim of the engineer is to create something useful. Characteristic of this difference is the mental approach to mathematical formulae expressing the relationship between various physical quantities. For the engineer a formula of this kind is generally little more than a convenient means of obtaining an unknown quantity from several known quantities; the form in which it is expressed is of little interest. The mathematician, on the other hand, places great value on the aesthetics of a formula, not merely for the sake of the aesthetics as such, but because he primarily views the formula as a relationship between various quantities in accordance with a certain law, which he endeavours to express in its neatest and simplest form.

Now it is interesting to discover that, in the everyday life of the engineer, there are many indispensable conceptions which were created by mathematicians, but have become an integral part of the everyday language of engineers. Among these are, for instance, the terms function, variable, parameter, and so on.

One of the great achievements of the philosopher Descartes was the discovery of analytical geometry, by which a link was forged between the geometrical conception and algebra. The very idea of expressing geometrical figures by algebraic formulae, on the one hand, and interpreting such formulae by geometry, on the other, was to have far-reaching consequences. Nowadays it is a matter of course for every engineer who has to perform a series of experiments to record the results, not merely as a table of figures, but also in the form of a graph, for which he utilizes the form of representation from which he can best obtain an empirical relationship between two or more measured quantities. In doing so he will begin by assuming that the desired quantity, termed mathematically as the dependent variable, varies in an unambiguous manner with respect to one or more other quantities, known as the independent variables. He will then perform a series of measurements in such a manner that for a particular group of readings only one dependent variable is allowed to change while all others are kept constant. He will attempt to plot the various readings belonging to one group in the form of a curve. Of course it will have to be assumed that the relationship in question is continuous, i.e. that a small change in one quantity does not lead to an abrupt change in the other. Often it is possible to pick out one of the dependent variables and to plot a complete family of curves on a single sheet of paper, on which the other variables now appear as parameters. He has thus succeeded in plotting the results of a long series of measurements on one graph which can be studied at a glance. These few remarks already give some idea of the tremendous amount of abstract thought which had to be carried out by the mathematicians of past centuries to enable this procedure to appear so self-evident nowadays.

Many other examples could be added, such as the use of complex numbers to represent alternating-current quantities for calculations in electrical engineering. Here we are not concerned with enumerating all possible examples, but rather with showing that even in technical spheres which appear to be extremely remote from mathematics, a certain amount of mathematical thought is involved somewhere.

Maxwell's Theory of Electromagnetism and the Early Days of Electrical Engineering

In the first half of the 19th century countless attempts were made by the mathematicians and physicists to summarize the numerous discoveries regarding electromagnetic phenomena by means of a self-contained mathematical theory. Not one of these attempts led to a conclusive success, and they are now all forgotten. It was Maxwell who succeeded in postulating the theory which found its final form in his now famous "treatise on electricity and magnetism" published in 1873. Aided by the experimental work of the renowned Faraday, with his lucid methods of illustration, he managed to summarize all the electromagnetic phenomena in four simple equations. If these are augmented by the three "laws of matter", i.e. providing the relationship between electrical field strength and dielectric displacement, or between magnetic field strength and flux density, together with Ohm's law, it is possible to explain all the phenomena known at that time, with the aid of this theory. The postulation of this theory is justly hailed as one of the greatest scientific achievements of all time. It is characteristic of a successful theory that, like Maxwell's, the equations not only satisfactorily explained electromagnetic phenomena known at the time they were postulated, but also permitted prognoses regarding phenomena which were still unknown. For example, Hertz discovered radio waves on the basis of predictions from Maxwell's equations, and the experimental discovery that the so-called electromagnetic constant is identical with the speed of light, led to the realization of the fact that optical phenomena involve electromagnetic waves.

A further feature of Maxwell's theory is its descriptiveness. The first two equations in their integral form permit of a concrete interpretation and are nowadays known to every high-school boy as the

law of induction and the law of magnetic flux ("Maxwell's Rule"). The two other equations express the fact that, firstly, the paths of electric current are always closed and, secondly, that the source of the electric field must be sought in the electric charge. The lucidity of this theory—consider for instance the terms inductance, capacitance, reluctance, leakage factor—is certainly one of the main reasons for the victorious advance made by electrical engineering at the end of the last century and the beginning of the present one. To understand the operation of rotating electrical machines and transformers there is no need for any special mathematical knowledge, and they can also be designed with simple formulae. It is therefore obvious that, at that time, intuition and imagination were more useful to the engineer than special mathematical knowledge. Even so, there were certain problems which necessitated intensive development of Maxwell's equations. One of the most notable is Rogowsky's calculation of the leakage field of a transformer. It is proof of Rogowsky's great mathematical skill that he should have succeeded in obtaining a final result, after considerable effort involving the expansion into Fourier series, leaving only one unexplained factor which is now known as the Rogowsky factor. Another phenomenon is the "skin effect" which causes the additional losses in the windings of electric machines. In this case, Field, starting from Maxwell's equations, postulated a very elegant theory, which allows these losses to be predetermined quite easily.

Modern Trends

Without exaggeration it may be claimed that mathematical knowledge and methods occupy a more important position in engineering than ever before. It is possible to give a number of reasons explaining this phenomenon, which will now be briefly outlined.

Naturally, the most obvious is the fact that nowadays, with electronic computers, it is possible to tackle problems which would formerly have been a hopeless task for an engineer equipped only with slide-rule or, at the best, a portable calculating machine. But it is a well-known fact that the electronic computers only execute the orders they are given; therefore suitable numerical methods are required to enable a computer to solve a complicated differential equation, for instance. Numerical analysis, as a branch of applied mathematics, has consequently witnessed a tremendous boom and fills an important place in the curriculum of universities.

It is remarkable that the need to make use of these new facilities is also felt in the typically classical spheres, such as heavy mechanical and electrical engineering. This is quite comprehensible if we consider the size of the machines involved nowadays. The enormous increase in the unit outputs of steam turbines, generators and transformers is not only due to technological progress; it would not have been feasible if the methods of calculation had not been improved at the same time. Only in this way has it been possible to achieve the double goal, of increasing the power-weight ratio while improving the reliability. It is quite obvious that, for instance, the mechanical problems associated with the construction of a 300-MVA generator need studying in far more detail than those for a machine with an output of the order of 10 MVA.

In the research programme of most engineering firms the study of transient phenomena of all kinds occupies an important position. In this field in particular the demands have increased very greatly. There was a time when it was regarded as a natural occurrence for a transformer to break down as the result of a lightning stroke. Nowadays it is taken for granted that a transformer will be so designed that it can withstand such stresses (provided, of course, it is equipped with a suitable means of overvoltage protection). Furthermore, it is remarkable that a multiple winding system, as represented by a transformer, possesses such a complex configuration that nobody has yet succeeded in establishing a theory for transients which proves satisfactory in all conditions.

Also in rotating machines transients, i.e. their behaviour when the load suddenly changes, are important. Naturally, generators were being built long before any theory existed regarding transients. Even today this theory has not been fully developed in all details. In this period of growing unit ratings and increasing interconnected operation, however, there is an obvious need for continuous refinement of the theory and methods of calculation.

The ability to perform extensive calculations quickly provides a notable incentive for theoretical work. Not so very long ago it was possible to speak of stagnation in this field. The basic operations were known in principle, and refinement of the theory was pointless, so long as there were not any numerical methods of utilizing it in practice. Compared with earlier eras, it is striking how short a time it now takes for new theoretical knowledge to become general knowledge among engineers. It may be recalled, for example, that Steinmetz and others recognized before the beginning of the present century that steady-state alternating-current quantities could be represented by complex numbers which could be employed for calculation. On the other hand, a text-book on electrical engineering[1] published in 1922, which had the reputation of being very up-to-date at that time, did not even mention this ability. Thus the present situation is stressed even more clearly.

What has been said so far in this article regarding spheres which may be regarded as classical, applies to an even greater extent to more recent technical creations. Whereas it might, perhaps, be feasible in an emergency to continue building generators and transformers without any improvement of the basic theory, or increased calculation, this is certainly not true of such fields as nuclear physics or guided missiles. Here the outlay for experimental development is so great that technical development is quite out of the question without well-founded mathematical theory and the facilities for carrying out extensive numerical calculations within a reasonable space of time.

[1] Theoretisches und praktisches Lehrbuch für Elektrotechniker. J. Fischer-Hinnen, Zurich 1922.

At this point it is certainly fitting to damp some of the enthusiasm regarding the capabilities of electronic computers. There are a great many problems whose numerical solution involves such a large number of operations that even the fastest electronic computers cannot manage them. We are mainly considering the solution of partial differential equations with more than two independent variables. Of practical importance among such problems is the calculation of three-dimensional fields where, even today, one cannot always be certain as to how these can be calculated within a reasonable space of time.

Finally it is worth saying a word about a branch of mathematics which has made great progress in the last few decades, namely statistics. The applications of this science continue to gain significance in engineering. The reason is obvious. Extended series of experiments, e.g. breakdown tests on special models, mostly take a long time and involve heavy costs. By means of statistical methods it is possible, with good advance planning, to gain a maximum of information with a minimum of experimental effort.

Conclusions

If we follow present technical developments attentively, we are bound to arrive at the conclusion that "mathematization" is making rapid progress in engineering. This implies that higher standards have to be set for the theoretical and mathematical training of engineers. But of course it does not mean that they can dispense with the main prerequisite for the creation of new technical products—an inventive imagination.

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THE INTEGRATION OF FUNCTIONS WITH THE AID OF ELECTRONIC COMPUTERS

681.14:517.3

For the evaluation of simple and multiple integrals the trapezoidal rule is the simplest method. However, it is seldom used because it is prone to introduce a much larger error than other methods, such as Simpson's formula. On the other hand, it offers the advantage that the calculation with half the integration step can be solved without difficulty, there being no need to maintain the functional values of the first calculation for the double step, or for them to be calculated again. Extrapolation in terms of h^2 according to Richardson (where h denotes the length of the step) yields the same results as Simpson's method when the trapezoidal rule is used twice in succession, with steps in the ratio of one to two. Thus the two methods are equivalent. By utilizing the Richardson extrapolation once more it is possible to determine whether the accuracy attained is adequate.

THE INTEGRATION of functions is one of the fundamental operations of mathematics. For the solution of many problems numerical integrals between wide limits are required, or multiple integrals, for example for the calculation of the forces produced by currents flowing in systems of conductors.

To attempt to undertake all the calculations needed is far beyond the scope of human capabilities, so that electronic computers have to be employed. For this it is necessary to employ definite methods, which also have to be tailored to suit the particular case of utilization with electronic computers. The question of the accuracy of calculation is closely related to the choice of the integration interval. In the course of this article these relationships will be illustrated by reference to an example.

Peculiarities of the Electronic Method of Calculation

The technique and capacity of numerical calculation was radically changed by the introduction of desk calculating machines. This applies to a much greater extent to the application of automatic computers.

An outstanding feature of the special methods used with electronic computers is their simplicity compared with some of the relatively complicated integral methods put forward by certain authors [1, 2, 3].

The following remarks are primarily intended to outline some of the advantages of the simpler methods, especially that using the trapezoidal rule:

$$\int_a^b \mathrm{f}\,\mathrm{d}x = \frac{b-a}{n}\left[\frac{1}{2}(y_0 + y_n) + \sum_{i=1}^{\overset{i=n-1}{\cancel{N-1}}} y_i\right] \qquad (1)$$

$$\text{with } y_i = \mathrm{f}\left(a + i\,\frac{b-a}{n}\right)$$

Above all, this formula is easily programmed; there is no need to store any constants in the computer. Also the number of integration steps n—within the total interval from a to b—enters into the calculation in a very simple manner. Particularly when a wide integration interval is involved, the constancy of the step length exerts a beneficial influence on the work of programming.

As will be known, the error is proportional to h^2, where $h = (b-a)/n$ denotes the integration step. In the next chapter a method will be demonstrated in which, by means of an artifice, it is possible to attain the same convergency, proportional to h^4, as is attainable with Simpson's formula.

The following reason is given for the preference shown to the trapezoidal rule. Assuming the integration were carried out with a step of length h, where $\sum \mathrm{f}$ is easy to keep in store, and the calculation were subsequently repeated with half this step length, only the values in the middle of each step of the first computation need to be calculated, the values so obtained rendering a contribution to those

determined with a step of half the length. This advantage is particularly marked when the calculation of f is tedious, especially when double integrals written in the form

$$\int_a^b \left(\int_{\varphi(x)}^{\psi(x)} f(x, y)\, dy \right) dx$$

have to be dealt with. This argument applies even more forcibly to multiple integrals.

Extrapolation in h^m

In 1927 Richardson put forward a method of improving the accuracy of numerical results, provided the order of magnitude of the error is known [4]. The basic idea of this method is easy to understand.

It may be assumed that, if h is sufficiently small, the error is proportional to h^m. Furthermore, the calculation is considered to have been performed with two values of h, e.g. with $h_1 = h$ and $h_2 = 2h$. If we desire to determine the exact value I, we must assume that the following values I_1 and I_2 are known in advance.

$$\begin{aligned} I_1 &= I + K\, h_1{}^m \\ I_2 &= I + K\, h_2{}^m \end{aligned} \qquad (2)$$

By eliminating K we obtain

$$I = \frac{\left(\dfrac{h_1}{h_2}\right)^m I_2 - I_1}{\left(\dfrac{h_1}{h_2}\right)^m - 1} \qquad (3)$$

A prerequisite condition for this result is that the relationship obeys equation (2), which applies for small values of h. The application of formula (3) is limited by round-off errors. Nevertheless it can be checked by calculation whether the equation (2) is fulfilled, by calculating I with (3) using different steps and comparing the results.

In the example given below the case is considered for $n = 4$ and $n = 2$, with $h_1 = 2\, h_2$; the latter assumption implies that each time the integration is performed with the step reduced by half, thereby facilitating the execution of the calculation.

For an operation with an error proportional to h^2 we obtain

$$I = \frac{4\, I_2 - I_1}{3} \qquad (3')$$

For an error of the order of magnitude of h^4

$$I = \frac{16\, I_2 - I_1}{15} \qquad (3'')$$

At an earlier stage it was mentioned that the trapezoidal rule yields an error of the order of h^2, hence the equation (3′) is valid.

The trapezoidal rule (1) is now applied to the integration with two different steps h and $2h$. Then, with $n = (b - a)/h$ it becomes

$$T_{2h} = \frac{b - a}{\left(\dfrac{n}{2}\right)} \left\{ \frac{1}{2}\, y_0 + y_2 + y_4 + \ldots + y_{n-2} + \frac{1}{2}\, y_n \right\}$$

$$T_h = \frac{b - a}{n} \left\{ \frac{1}{2}\, y_0 + y_1 + y_2 + y_3 + y_4 + \ldots \right.$$
$$\left. \ldots + y_{n-2} + y_{n-1} + \frac{1}{2}\, y_n \right\}$$

From equation (3′) we obtain

$$I = \frac{b - a}{3\, n} \{ y_0 + 4\, y_1 + 2\, y_2 + 4\, y_3 + 2\, y_4 + \ldots$$
$$\ldots + 2\, y_{n-2} + 4\, y_{n-1} + y_n \} \qquad (4)$$

This form is identical with Simpson's formula, for which the error is known to be of the order of h^4. Supposing S_h is the result of the computation with the step h, then

$$S_h = \frac{4\, T_h - T_{2h}}{3} \qquad (4')$$

It is also possible for the values obtained from Simpson's formula with the partial interval h to be extrapolated with h^m, as follows:

$$I = \frac{16\, S_h - S_{2h}}{15} \qquad (5)$$

From this aspect extrapolation in h^m automatically allows the length of the step to be controlled. Another possibility is afforded by the iteration of S_{2h}, S_h, $S_{h/2}$ and calculating I each time. If, in the course of this, the difference between two successive values of I is

less than a given amount ε, the last value obtained is regarded as an approximation for the desired integral.

The procedure is as follows:

Starting with $h = (b - a)/2$, the following values are calculated with the aid of the trapezoidal rule, in that the number of steps in the integration is doubled each time. If t_n denotes the value obtained with 2^n intervals, we obtain for $n \geqslant 4$

$$I_n = \frac{64\, t_n - 20\, t_{n-1} + t_{n-2}}{45} \tag{5'}$$

This formula corresponds to (5) in which, with the aid of (4′), the value given by Simpson's formula was expressed by those obtained with the trapezoidal rule. The calculation is discontinued when $I_n - I_{n-1}$ is smaller than ε.

Example

The starting point for the above study was the consideration that it would be illogical in the case of a wide integration interval for points situated within the limits to be differently weighted. Thus, with the previous notation, the following formula is obtained

$$\int_a^b f\,dx = \frac{b-a}{n}\left\{\frac{n}{2n+1}(y_0 + y_n) + \frac{n^2}{n^2-1}\sum_{i=1}^{n-1} y_i\right\} \tag{6}$$

This formula provides accurate results for polynomial expressions of the third order. Since it is extremely complicated to estimate the error, evaluation is replaced by an experimental method

Supposing, for example, $\int_0^{10} \sin x\,dx = 1{\cdot}8390715$ were to be calculated.

In Table I the values obtained by the three methods given above—trapezoidal rule, Simpson's formula and equation (6)—are tabulated to the seventh place of decimals, enabling them to be compared.

It will be recognized at once that Simpson's formula yields more accurate results, i.e. smaller error, than the other two methods. The order of magnitude of the error can be estimated as follows. Supposing I is the exact result and, with the aid of the trapezoidal rule, we obtain

$$I = t_n + K\,h^2 \quad \text{where } h = (b - a)/n$$

and
$$I = t_{n+1} + \frac{K}{4}\,h^2$$

Eliminating K we obtain

$$\frac{I - t_n}{I - t_{n+1}} = 4$$

For a formula of the order h^m the relationship between two successive differences, in the limiting case, is 2^m.

Table II shows the ratios of successive errors in terms of n.

TABLE I

Number of partial intervals	Formula (6)		Simpson's formula		Trapezoidal rule	
	Value	Error*	Value	Error*	Value	Error*
2	−7·2995303	91386018	−7·2995303	91386018	−6·1546741	79937456
4	0·9961065	8429650	3·0700157	12309442	0·7638433	10752282
8	1·6615958	1774757	1·8695611	304896	1·5931317	2459398
16	1·7964553	426162	1·8407061	16346	1·7788125	602590
32	1·8285228	105487	1·8391702	987	1·8240808	149907
64	1·8364410	26305	1·8390778	63	1·8353285	37430
128	1·8384144	6571	1·8390721	6	1·8381362	9353
256	1·8389074	1641	1·8390717	2	1·8388378	2337
512	1·8390306	409	1·8390717	2	1·8390132	583
1024	1·8390614	101	1·8390717	2	1·8390570	145

* Given as absolute values $\times 10^7$

TABLE II

n	Trapezoidal rule	Formula (6)	Simpson's formula
4	7·4344642	10·841022	7·4240585
8	4·371916	4·7497488	40·3726
16	4·081349	4·16451	18·653
32	4·01976	4·03994	16·56
64	4·0050	4·0102	15·7
128	4·0019	4·0032	—
256	4·002	4·012	
512	4·02	4·05	

TABLE III

n	Formula (5′)	Simpson's formula
8	1·789531	1·869561
16	1·828782	1·840706
32	1·839068	1·839170
64	1·839072	1·839078
128	1·839072	1·839072
Exact result: 1·839072		

From the above Table II it is evident that the quotients tend towards a limit, the round-off error becoming increasingly pronounced. For both the trapezoidal rule and the formula in equation (6) it attains the limit 4. This indicates that for both methods of calculation the error possesses the order of magnitude of h^2. The theoretical proof of this fact, however, encounters considerable difficulties.

In conclusion the trapezoidal rule is now compared to the expression (5′). Table III shows an improvement on Simpson's formula, in that it manages with one iteration less.

Conclusions

The examples listed in the Tables I to III yield the following results.

Formula (6) should not be used as it does not provide any better results than those obtained with the trapezoidal rule. Experience confirms the correct estimation of the error, thus justifying the use of the suggested method of calculation. Hence useful results may be expected with this method.

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THE USE OF DIGITAL COMPUTERS TO CALCULATE BESSEL AND ALLIED FUNCTIONS

681.14:517.5

The article gives the polynomial approximations for modified Bessel and Struve functions and their integrals for the parameters 0 and 1 in the interval $0 \leqslant x < \infty$, which are often required for the computation of axially symmetrical problems such as those connected with the magnetic fields on transformers with concentric windings.

FOR THE AUTOMATIC calculation of transcendent functions by electronic computers, the best expressions from the programming aspect are usually provided by cut-off power series or asymptotic series. For large arguments, though, power series may be numerically unfavourable, owing to the round-off error. On the other hand, sometimes the asymptotic series cannot be used with the same x values (e.g. for $K_0(x)$), because the accuracy thereby obtained is inadequate. Generally speaking, another method is more appropriate. In that case polynomials of the form $\overline{P}_n(x)$ are employed which afford a close enough approximation to the function $f(x)$ in a given interval (x_0, x_1). Thus for these polynomials

$$\operatorname*{Max}_{x_0 \leqslant x \leqslant x_1} |f(x) - \overline{P}_n(x)|$$

should be smaller than

$$\operatorname*{Max}_{x_0 \leqslant x \leqslant x_1} |f(x) - P_n(x)|$$

in which $P_n(x)$ is any polynomial whose order does not exceed n. The polynomial $\overline{P}_n(x)$ is known as the Chebyshev approximation of the n-th order for $f(x)$. A feature of it is that the maxima, of which there are not more than $n+2$, for the expression

$$|f(x) - \overline{P}_n(x)|$$

are all equal in the interval (x_0, x_1) [1, 2].

Since the calculation of the coefficients of $\overline{P}_n(x)$ is extremely tedious, a number of numerical methods have been devised, which do not yield the Chebyshev approximation itself, but provide polynomials of almost the same accuracy. Two of these methods, which have given satisfactory results, will be utilized in the following remarks. Other feasible methods, such as the one utilizing rational approximations, as propounded by Maehly [4, 5], are not considered.

During the calculation of the magnetic field of transformers with concentric windings it became necessary to calculate Bessel and allied functions, for which great accuracy was stipulated, and the shortest possible time allowed for calculation. For this task auxiliary polynomials were determined; with their aid and the elementary functions e^x, $\log_e x$ and $\sqrt{x}$, it is possible to calculate the modified Bessel functions I_0, I_1, K_0, K_1, $I_0 - L_0$, $I_1 - L_1$ (L being the modified Struve functions), as well as the integrals $\int x\, I_1\, dx$, $\int x\, K_1\, dx$, $\int x(I_1 - L_1)\, dx$. The accuracy of the auxiliary polynomials over the entire interval $0 \leqslant x \leqslant \infty$, amounts to nine significant digits.

Method of Calculation

The polynomial approximations for

$$K_0,\ K_1,\ \int x\, K_1\, dx$$

were determined, using the Lanczos τ method [1, 3]. Attempts to adopt the same method for

$$I_0,\ I_1,\ \int x\, I_1\, dx,\ I_0 - L_0,\ I_1 - L_1,\ \int x(I_1 - L_1)\, dx$$

proved less successful, so that, for these functions, approximations according to the telescoping method were calculated.

The τ Method

It is assumed that the function $y = f(x)$ is given as the solution of a linear differential equation $L(y) = 0$. The coefficients are assumed to be polynomials in x. Lanczos's τ method now permits a convergent series of polynomials of increasing order to be formed, in which the accuracy improves with the order of the approximation polynomial.

For this purpose the equation $L(y) = 0$ is converted so that a term $\tau T_n(x)$ has to be added on the right-hand side. τ is a small, constant number and $T_n(x)$ is the Chebyshev polynomial of the n-th order in the interval (0,1). The resultant, modified equation

$$\bar{L}(y) \equiv L(y) - \tau T_n(x) = 0$$

then yields a polynomial as solution, and this exact solution of the modified differential equation is an approximation for $f(x)$ in the interval (0,1) with practically the same accuracy as the Chebyshev approximation polynomial of the same order.

The τ method often also yields approximations in the interval (a, ∞) in those cases where the asymptotic series is inapplicable; on the other hand, it is not always possible to predict whether the method will yield usable results or not.

An example will now be shown in which the method is employed twice in succession. In order to obtain a series of convergent polynomials in the interval $1 < x < \infty$ for the function

$$\pi/2 - \int_0^x \xi\, K_1(\xi)\, d\xi \quad [1]$$

it is necessary to start from the asymptotic form of the function $K_1(x)$

$$K_1(x) = \frac{e^{-x}}{\sqrt{x}}\left(e_0 + \frac{e_1}{x} + \ldots\right) = \frac{e^{-x}}{\sqrt{x}} \cdot z\left(\frac{1}{x}\right)$$

The function $z(1/x)$ then satisfies the differential equation

$$4\left(\frac{1}{x}\right)^2 \frac{d^2 z\left(\frac{1}{x}\right)}{d\left(\frac{1}{x}\right)} + 8\left[\frac{1}{x} + 1\right] \frac{dz\left(\frac{1}{x}\right)}{d\left(\frac{1}{x}\right)} - 3z\left(\frac{1}{x}\right) = 0 \qquad (1)$$

[1] The constant is chosen so that the function tends towards 0 when $x \to \infty$ [8].

obtained from the differential equation for $K_1(x)$ when $\frac{e^{-x}}{\sqrt{x}} z\left(\frac{1}{x}\right)$ is substituted therein for $K_1(x)$.

With the formal development

$$z_n\left(\frac{1}{x}\right) = b_0 + \frac{b_1}{x} + \ldots + \frac{b_n}{x^n} \qquad (2)$$

and adding $\tau_1 T_n(1/x)$ to the right-hand side, the following recurrence formula is obtained for the values of b_k ($k = n, \ldots 0$)

$$b_k = \frac{\tau_1/t_{n,k} - 8(k+1)\, b_{k+1}}{(2k+1)^2 - 4} \qquad (3)$$

in which $t_{n,k}$ is the coefficient of the k-th power of $1/x$ in the polynomial $T_n(1/x)$.

With the supplementary condition

$$b_{n+1} = 0$$

which follows from (2), and the further condition

$$b_0 = \sqrt{\pi/2}$$

resulting from the boundary condition for $K_1(x)$ at infinity, the coefficients b_k can be unambiguously calculated from (3). The above supplementary condition $b_{n+1} = 0$ also fixes τ. This method can be used when τ becomes sufficiently small for a definite value of n.

The most obvious asymptotic series which can be used to represent the function

$$\pi/2 - \int_0^x \xi\, K_1(\xi)\, d\xi$$

is obtained by partially integrating the asymptotic expansion of the function $x K_1(x)$, term by term, taking into account the boundary conditions

$$\pi/2 - \int_0^x \xi\, K_1(\xi)\, d\xi = e^{-x}\sqrt{x}\left(c_0 + \frac{c_1}{x} + \ldots\right) = e^{-x}\sqrt{x} \cdot y\left(\frac{1}{x}\right)$$

The function $y(1/x)$ then satisfies the differential equation

$$2\left(\frac{1}{x}\right)^2 \frac{dy\left(\frac{1}{x}\right)}{d\left(\frac{1}{x}\right)} + \left[2 - \left(\frac{1}{x}\right)\right] y\left(\frac{1}{x}\right) - 2z\left(\frac{1}{x}\right) = 0 \qquad (4)$$

when $z(1/x)$ is the function defined by (1).

With an expression for $y_{n-1}(1/x)$, analogous to that for $z_n(1/x)$ in equation (2)

$$y_{n-1}\left(\frac{1}{x}\right) = a_0 + \frac{a_1}{x} + \ldots + \frac{a_{n-1}}{x^{n-1}}$$

a further recurrence formula is obtained for the coefficients a_k (where $k = n-1, \ldots, 0$) from (4), a term $\tau_2\, T_n(1/x)$ having also been added to the right-hand side. This formula is

$$a_k = \frac{\tau_2\, t_{n,k+1} - 2\, a_{k+1} - 2\, b_{k+1}}{2\, k - 1} \qquad (5)$$

Since the conditions

$$a_n = 0 \quad \text{and} \quad a_0 = \sqrt{\frac{\pi}{2}}$$

apply here too, and the values of b_k are known from (3), it is possible to calculate the coefficients a_k unambiguously.

Telescoping Method

Supposing a power series

$$P_n(x) = a_0 + a_1\, x + \ldots + a_n x^n \qquad (6)$$

is given, which represents the function $y = f(x)$ sufficiently accurately in the interval (0, 1), the last coefficient a_n should not be below the limit of the accuracy, i.e. the term $a_n x^n$ is not so small that it can be neglected. The telescoping of $P_n(x)$ is now based on the approximation of all powers of x by polynomials of lower order, but with practically the same accuracy. The coefficient $t_{n,n}$ in the Chebyshev polynomial

$$T_n(x) = t_{n,n}\, x^n + t_{n,n-1}\, x^{n-1} + \ldots + t_{n,0}$$

is equal to 2×4^{-n}, so that

$$\frac{T_n(x)}{2^{2n-1}} = x^n + \alpha_{n-1}\, x^{n-1} + \ldots + \alpha_0$$

or, also,

$$x^n = -\,(\alpha_{n-1}\, x^{n-1} + \ldots + \alpha_0) + \frac{T_n(x)}{2^{2n-1}}$$

Thus if we substitute

$$-\,(\alpha_{n-1}\, x^{n-1} + \ldots + \alpha_0)$$

for x^n in (6), we introduce an error which, in the entire interval (0, 1), at no time exceeds the value of 2×4^{-n}, and the calculation of $P_n(x)$ is reduced to that of a polynomial $P_{n-1}(x)$ of the $(n-1)$-th order. This process of telescoping can now be repeated until the last coefficient a_m is so large that even multiplying it by 2×4^{-m} cannot diminish it enough. The telescoping process is then finished and the original series, an approximation to the Chebyshev approximation, determined.

Approximation Polynomials for the Modified Bessel and Struve Functions and their Integrals

The approximations given below have been calculated in such a way that, for the interval $(0, a)$, the corresponding power series were improved by the telescoping method. For the interval (a, ∞) either the corresponding asymptotic series were truncated, also by the telescoping method, or the τ method was utilized to obtain the approximations. In the process, attention was paid to preventing the error from exceeding five units of the tenth significant digit over the whole interval $(0, \infty)$.

1. $I_0(x) = I_0$

$$0 \leq x \leq 10 \qquad I_0 = \sum_k a_k \left(\frac{x}{10}\right)^{2k} + \varepsilon$$

$$10 \leq x < \infty \qquad I_0 = \frac{e^x}{\sqrt{x}} \left[\sum_k b_k \left(\frac{10}{x}\right)^k + \varepsilon\right]$$

	a_k	b_k
ε_{max}	0·00000 0003	0·00000 00002
$k=0$	1·00000 00027	0·39894 22804 7
1	24·99999 90983	0·00498 67722 4
2	156·25005 02179	0·00028 06006 0
3	434·02668 00860	0·00002 86977 0
4	678·18085 14731	0·00000 58039 7
5	678·08454 24100	−0·00000 07275 3
6	471·31196 73418	0·00000 10787 1
7	239·23918 92805	
8	95·90195 23737	
9	26·24210 75769	
10	9·66646 25536	
11	0·14399 45401	
12	0·66883 15150	

2. $I_1(x) = I_1$

$$0 \leq x \leq 10 \qquad I_1 = x\left[\sum_k a_k \left(\frac{x}{10}\right)^{2k} + \varepsilon\right]$$

$$10 \leq x < \infty \qquad I_1 = \frac{e^x}{\sqrt{x}} \left[\sum_k b_k \left(\frac{10}{x}\right)^k + \varepsilon\right]$$

	a_k	b_k
ε_{max}	0,00000 00001	0,00000,00002
$k=0$	0·50000 00000 9	0·39894 22803 2
1	6·24999 99695 0	–0·01496 03283 8
2	26·04166 83656 6	–0·00046 76177 3
3	54·25343 50705 6	–0·00004 03143 9
4	67·81726 17980 4	–0·00000 72622 7
5	56·51119 24416 1	0·00000 07463 2
6	33·65156 77364 2	–0·00000 12354 5
7	14·98220 50575 7	
8	5·28380 36833 2	
9	1·35566 93198 1	
10	0·41187 05935 5	
11	0·01604 45909 7	
12	0·02411 17429 2	

3. $\int\limits_0^x \xi I_1(\xi)\,\mathrm{d}\xi = i_1$

$$0 < x < 19 \qquad i_1 = x^3\left[\sum_x a_k\left(\frac{x}{19}\right)^{2k} + \varepsilon\right]$$

$$19 \leqslant x < \infty \qquad i_1 = \mathrm{e}^x\sqrt{x}\left[\sum_k b^k\left(\frac{19}{x}\right)^k + \varepsilon\right]$$

	a_k	b_k
ε_{max}	0·00000 00000 5	0·00000 00003
$k=0$	0·16666 66666 97	0·39894 22804 2
1	4·51249 99781 47	–0·01837 23441 5
2	48·48251 74891 78	–0·00061 29416 9
3	283·60014 30398 28	–0·00005 46931 2
4	1047·06815 64989 18	–0·00000 64035 4
5	2665·27067 48766 90	–0·00000 36450 6
6	4964·09851 71536 51	0·00000 17385 3
7	7054·95843 37915 41	–0·00000 09470 5
8	7933·29573 08716 29	
9	7116·15292 07114 55	
10	5571·34195 40440 09	
11	2960·52407 13922 96	
12	2532·12381 56837 46	
13	–94·65220 30647 81	
14	1245·53338 57226 89	
15	–533·52266 96117 54	
16	366·76444 46018 50	
17	–97·36153 67903 26	
18	21·24575 56155 02	

4. $K_0(x) = K_0$

$$0 < x \leqslant 1 \qquad K_0 = \left(\gamma + \log_e \frac{x}{2}\right)\cdot$$

$$\cdot\left[\sum_k a_k x^{2k} + \varepsilon\right] + x^2\left[\sum_k b_k x^{2k} + \varepsilon\right]$$

$$\gamma = 0{\cdot}57721\ 56649\ 01533$$

$$1 < x < \infty \qquad K_0 = \frac{\mathrm{e}^{-x}}{\sqrt{x}}\left[\sum_k c_k\left(\frac{1}{x}\right)^k + \varepsilon\right]$$

	a_k	b_k	c_k
ε_{max}	0·00000 00007	0·00000 00000 1	0·00000 0002
$k=0$	–1·00000 00001 3	0·25000 00000 02	1·25331 41373
1	–0·24999 99933 8	0·02343 74998 87	–0·15666 40293
2	–0·01562 50529 8	0·00479 57184 94	0·08809 47319
3	–0·00043 38794 3	0·00001 41259 84	–0·09099 77576
4	–0·00000 69512 3	0·00000 01577 33	0·13035 95356
5			–0·20650 10623
6			0·29703 31052
7			–0·33905 07775
8			0·28091 80905
9			–0·15577 95499
10			0·05128 12921
11			–0·00754 46371

5. $K_1(x) = K_1$

$$0 < x \leqslant 1 \qquad K_1 = \frac{1}{x} + \left(\gamma + \log_e \frac{x}{2}\right)\cdot$$

$$\cdot\left[\sum_k a_k x^{2k} + \varepsilon\right] + x\left[\sum_k b_k x^{2k} + \varepsilon\right]$$

$$\gamma = 0{\cdot}57721\ 56649\ 01533$$

$$1 < x < \infty \qquad K_1 = \frac{\mathrm{e}^{-x}}{\sqrt{x}}\left[\sum_k c_k\left(\frac{1}{x}\right)^k + \varepsilon\right]$$

	a_k	b_k	c_k
ε_{max}	0·00000 00000 5	0·00000 00000 9	0·00000 0002
$k=0$	0·50000 00000 22	–0·25	1·25331 41373
1	0·06249 99988 96	–0·07812 5	0·46999 25204
2	0·00260 41754 97	–0·00434 02777	–0·14683 86549
3	0·00000 42287 47	–0·00010 62463 83	0·12757 69501
4	0·00000 07064 25	–0·00000 14806 68	–0·16879 46725
5		–0·00000 00133 75	0·25702 30979
6			–0·36241 23971
7			0·40933 35460
8			–0·33710 41487
9			0·18623 64530
10			–0·06115 49277
11			0·00898 15841

6. $\pi/2 - \int_0^x \xi\, K_1(\xi)\, d\xi = k_1$

$0 < x < 1 \quad k_1 = \pi/2 - x + x^3 \left(\gamma + \log_e \frac{x}{2}\right) \cdot$

$$\cdot \left[\sum_k a_k\, x^{2k} + \varepsilon\right] + x^3 \left[\sum_k b_k\, x^{2k} + \varepsilon\right]$$

$$\gamma = 0{\cdot}57721\ 56649\ 01533$$

$1 \leqslant x < \infty \quad k_1 = e^{-x} \sqrt{x} \left[\sum_k c_k \left(\frac{1}{x}\right)^k + \varepsilon\right]$

	a_k	b_k	c_k
ε_{max}	0·00000 00000 05	0·00000 00000 08	0·00000 0002
$k = 0$	–0·16666 66666 675	0·13888 88888 910	1·25331 41373
1	–0·01249 99999 575	0·01812 49998 963	1·09664 88851
2	–0·00037 20241 492	0·00067 31867 710	–0·69509 10682
3	–0·00000 60272 126	0·00001 24726 259	1·16661 91735
4	–0·00000 00627 385	0·00000 01428 666	–2·99524 18887
5			9·43322 66362
6			–30·61340 88398
7			90·41176 79352
8			–226·05985 97708
9			460·46962 68423
10			–747·44666 97296
11			951·10275 99993
12			–932·46524 38657
13			688·19980 22368
14			–369·07796 01792
15			135·57552 40815
16			–30·47957 22146
17			3·16059 58754

7. $I_0(x) - L_0(x) = S_0$

$0 < x < 20{\cdot}25 \qquad S_0 = \sum_k a_k \left(\frac{x}{20{\cdot}25}\right)^k + \varepsilon$

$20{\cdot}25 < x < \infty \qquad S_0 = \frac{1}{x}\left[\sum_k b_k \left(\frac{20{\cdot}25}{x}\right)^{2k} + \varepsilon\right]$

	a_k	b_k
ε_{max}	0·00000 00000 4	0·00000 00007
$k = 0$	0·99999 99999 65	0·63661 97726 9
1	–12·89155 03526 05	0·00155 24793 6
2	102·51561 81809 52	0·00003 41867 7
3	–587·37077 45667 79	0·00000 17952 6
4	2627·34459 14092 55	0·00000 05069 7
5	–9633·91509 95177 90	
6	29920·29683 84453 68	
7	–80546·90721 62114 48	
8	1 91082·95533 82325 02	
9	–4 03861·79309 34627 77	
10	7 64503·93519 33666 21	
11	–12 95216·38659 23775 26	
12	19 50764·97690 44749 25	
13	–25 81260·86233 18439 89	
14	29 54256·88282 76632 76	
15	–28 71496·73890 00573 20	
16	23 21565·07609 97806 52	
17	–15 23703·71975 31415 93	
18	7 87453·31186 20749 97	
19	–3 07281·58366 25414 90	
20	84863·78832 02513 07	
21	–14751·57081 27945 28	
22	1211·58770 94691 98	

8. $I_1(x) - L_1(x) = S_1$

$0 < x < 19 \qquad S_1 = x\left[\sum_k a_k \left(\frac{x}{19}\right)^k + \varepsilon\right]$

$19 < x < \infty \qquad S_1 = \frac{2}{\pi} - \frac{1}{x^2}\left[\sum_k b_k \left(\frac{19}{x}\right)^{2k} + \varepsilon\right]$

	a_k	b_k
ε_{max}	0·00000 00000 3	0·00000 03
$k = 0$	0·49999 99999 79	0·63661 9700
1	–4·03192 52059 27	0·00529 2639
2	22·56249 71324 95	0·00020 9945
3	–97·03482 85493 96	0·00003 4520
4	339·37212 81780 86	
5	–1000·73942 09260 89	
6	2550·99133 94458 52	
7	–5721·81554 06125 88	
8	11426·34647 35352 31	
9	–20429·85677 72348 26	
10	32659·60240 77395 51	
11	–46278·37659 26248 03	
12	57253·74660 14571 80	
13	–60622·87018 55409 99	
14	53665·13699 96938 17	
15	–38656 42330 24412 35	
16	21927·84744 97397 40	
17	–9377·30090 93586 30	
18	2830·60011 95461 30	
19	–536·00645 34299 52	
20	47·78333 21376 74	

9. $\int\limits_0^x \xi\,[I_1(\xi) - L_1(\xi)]\,\mathrm{d}\xi = s_1$

$$0 < x < 17 \quad s_1 = x^3 \left[\sum_k a_k \left(\frac{x}{17} \right)^k + \varepsilon \right]$$

$$17 < x < \infty \quad s_1 = c + \frac{x^2 - \log_e x^2}{\pi} + \frac{1}{x^2} \left[\sum_k b_k \left(\frac{17}{x} \right)^{2k} + \varepsilon \right]$$

$$c = -0{\cdot}17211\,83100$$

	a_k	b_k
ε_{max}	0·00000 00000 1	0·00005
$k = 0$	0·16666 66666 59	0·95495 205
1	–0·90187 80049 57	0·02441 489
2	3·61249 92739 70	0·00282 340
3	–11·58408 64250 65	
4	31·07087 55157 04	
5	–71·72406 60156 94	
6	145·34806 20139 04	
7	–262·08507 46701 04	
8	423·48982 78720 85	
9	–612·78795 39166 28	
10	786·34127 71923 36	
11	–878·39229 00548 06	
12	832·55508 27743 51	
13	–649·04359 94610 06	
14	401·01836 81777 91	
15	–187·33116 59739 68	
16	61·77872 28094 79	
17	–12·76042 97736 49	
18	1·23748 46452 04	

Conclusion

The polynomial coefficients given in the foregoing enable all nine of the functions

$$I_0, \quad I_1, \quad \int x\,I_1\,\mathrm{d}x, \quad K_0, \quad K_1, \quad \int x\,K_1\,\mathrm{d}x,$$
$$I_0 - L_0, \quad I_1 - L_1, \quad \int x\,(I_1 - L_1)\,\mathrm{d}x$$

to be solved to an accuracy of nine significant digits. The advantage over polynomial approximations published elsewhere [9] is the accuracy obtained; furthermore, the approximations for

$$\int x\,I_1\,\mathrm{d}x, \quad \int x\,K_1\,\mathrm{d}x, \quad I_0 - L_0, \quad I_1 - L_1, \quad \int x\,(I_1 - L_1)\,\mathrm{d}x$$

are new.

For practical applications, such as calculating the magnetic fields of transformers with concentric windings, which have to be performed on a routine basis in the Company, as well as for axially symmetrical problems encountered in nuclear engineering [7], the accuracy afforded has proved a great relief to the calculators.

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NON-LINEAR PLANE MAGNETIC FIELDS

538.122:530.182

Numerous examples of linear magnetic fields are known. This article examines some of the non-linear, i.e. the saturated fields. For a certain class of magnetization curves some fields can be calculated exactly, as shown.

Linear and Non-Linear Magnetic Fields

MAGNETIC FIELDS are said to be linear when the flux density B is proportional to the field strength H, that is to say when $B = \mu H$, the permeability μ being constant. This condition obtains when the magnetic field runs in air. If the lines of force run in parallel planes, the field is said to be plane. Such magnetic fields have been thoroughly dealt with in the literature. A particularly suitable means of calculating them is the theory of functions. For experimental investigations the electrolytic tank may be used. There is also an extensive literature on this subject.

In contrast to fields in air, those in iron are subject to saturation. They are therefore non-linear and cannot be determined by the above methods. In the literature on the subject of saturation the main stress is laid on the magnetization curves. In this article, though, it is proposed to examine a few non-linear fields.

These depend, not only on the geometrical form of their boundaries, but also on the magnetization curve. The present investigation is confined to a certain class of these curves, namely those conforming to the equation $H = cB^n$, where n is a fixed, real power. The measured curves approximate closely to this expression over a wide range. With this formula the problem is exactly soluble for a number of cases, such as axially symmetrical fields, for fields in a parallel strip, for the field in an angle, and for the plane dipole field. The calculated lines of force are illustrated graphically.

The Magnetization Curves

As mentioned above, the calculations are confined to the magnetization curve $H = cB^n$. By employing a suitable scale for H and B it is possible to make $c = 1$. This formula is chosen for the following reasons. If the flux density B is increased by a constant factor k, the field strength H, in the case of a single power, increases by the positionally independent factor k^n. The lines of force and potential retain their shape. With other magnetization curves, however, the formation of the lines of force changes with the intensity. This limitation considerably simplifies the calculation in many cases. The field illustrated in the third chapter can be calculated for any desired curve, but this does not appear to apply to the other cases dealt with.

The assumed expression also exhibits a remarkable symmetry. If we substitute $1/n$ for n, and H for B, the equation of the magnetization curve is in effect unaltered. In the diagram the lines of flux and potential are interchanged. This fact can be utilized to obtain the potential lines from the formula giving the flux lines.

If the flux density B and the field strength H are both plotted to a logarithmic scale, the idealized magnetization curve becomes a straight line. Fig. 1 shows a measured curve. In the range from about 11 to 20 kGs for the flux density, corresponding to an interval from roughly 3 to 300 A/cm of the field strength H, the curve is approximately straight. This is the most important range for practical applications. Higher flux densities are seldom used and at low values the field strength is so weak that the absolute errors can be tolerated in the majority of cases. This shows that the above formula is a sufficiently good approximation for practical purposes.

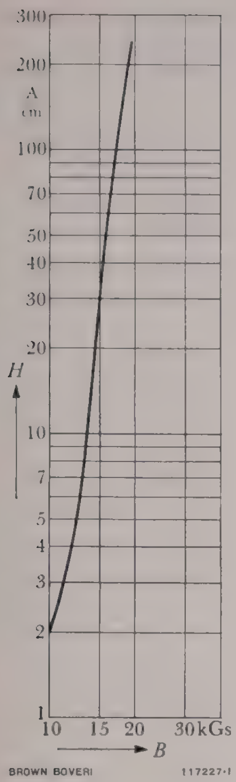

Fig. 1. – Magnetization curve

The flux density B and the field strength H are plotted to a logarithmic scale. The curve is approximately a straight line with the gradient 9.

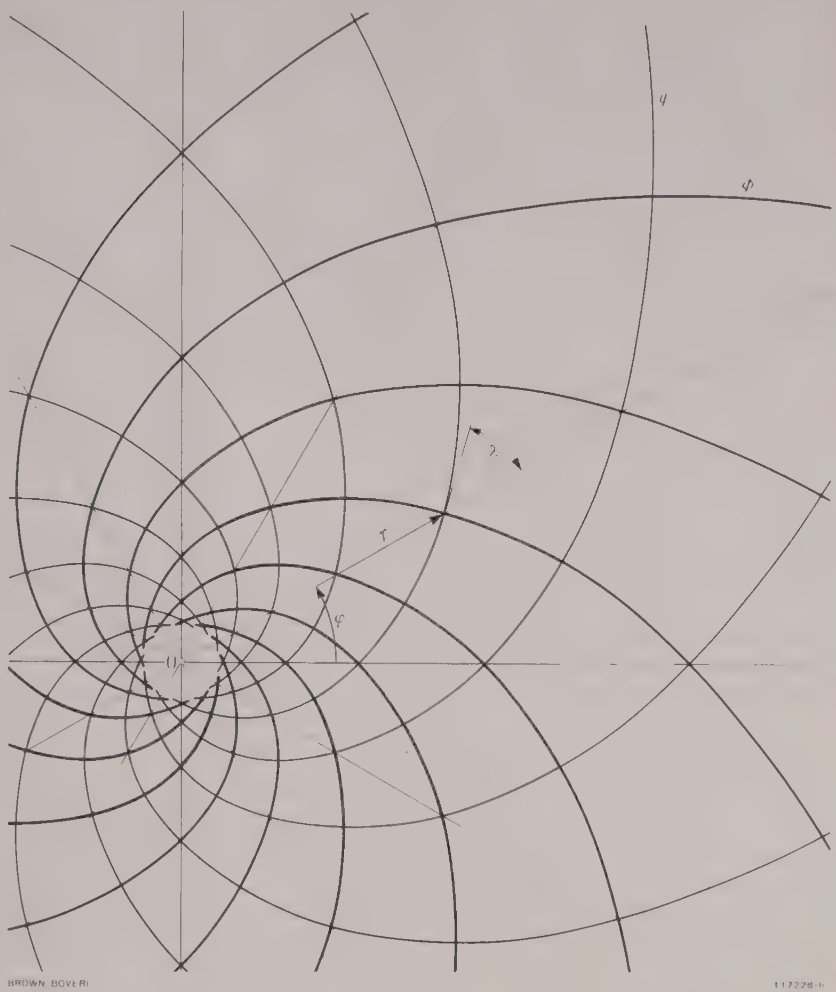

Fig. 2. – Linear, axially symmetrical, spiral field, plotted for $\lambda = 45°$

Ψ = Flux lines
Φ = Potential lines
r, φ = Polar co-ordinates
λ = Angle between radius vector and field vector

The powers n are obtained from the gradient of the straight lines. For the numerical calculation $n = 9$ was chosen. The adoption of $n = 1$ reverts to the linear case.

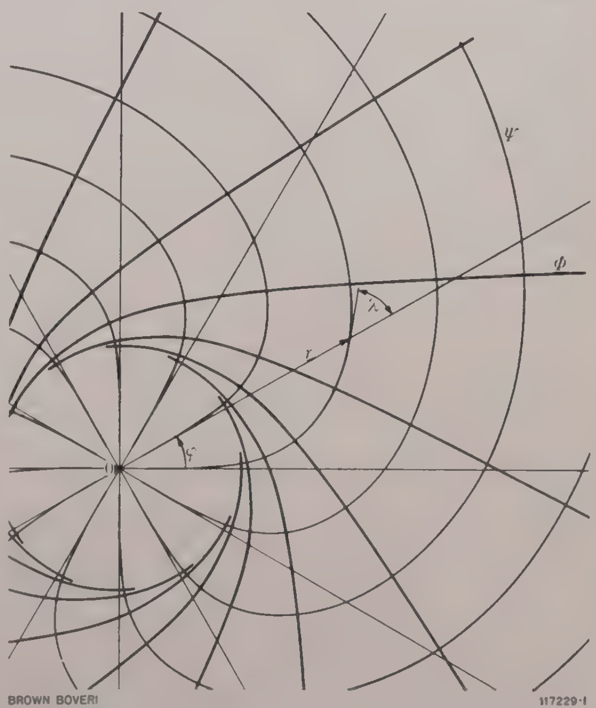

Fig. 3. – Non-linear, axially symmetrical, spiral field

Notation as for Fig. 2.

Axially Symmetrical Fields

A field of this kind is composed of an eddy and a source. On account of the eddy, the magnetic potential increases by an amount H_0, independent of the radius r, in one revolution round the centre of rotation. Thus the azimuthal field strength is $H_\varphi = H_0/2\pi r$. If the source has an intensity of B_0, at the radius r the radial component of the flux density is given by $B_r = B_0/2\pi r$.

If λ is used to denote the angle between the radius vector and the vector of the field strength, $H_\varphi = H \sin\lambda$ and $B_r = B\cos\lambda$. Taking B and H from the above equations and substituting them in the formula for the magnetization curve, we obtain

$$r^{n-1} = K\,\frac{\sin\lambda}{\cos^n\lambda}$$

in which K is a constant. In the linear case ($n = 1$), λ is independent of r. The lines of force are logarithmic spirals. In the non-linear magnetic field ($n > 1$), $|\lambda|$ grows with r. For very small values of r the lines of force are almost radial. For larger values of r they are bent and tend to run almost azimuthally. The lines of force in the linear field are shown in Fig. 2, those of a non-linear field in Fig. 3.

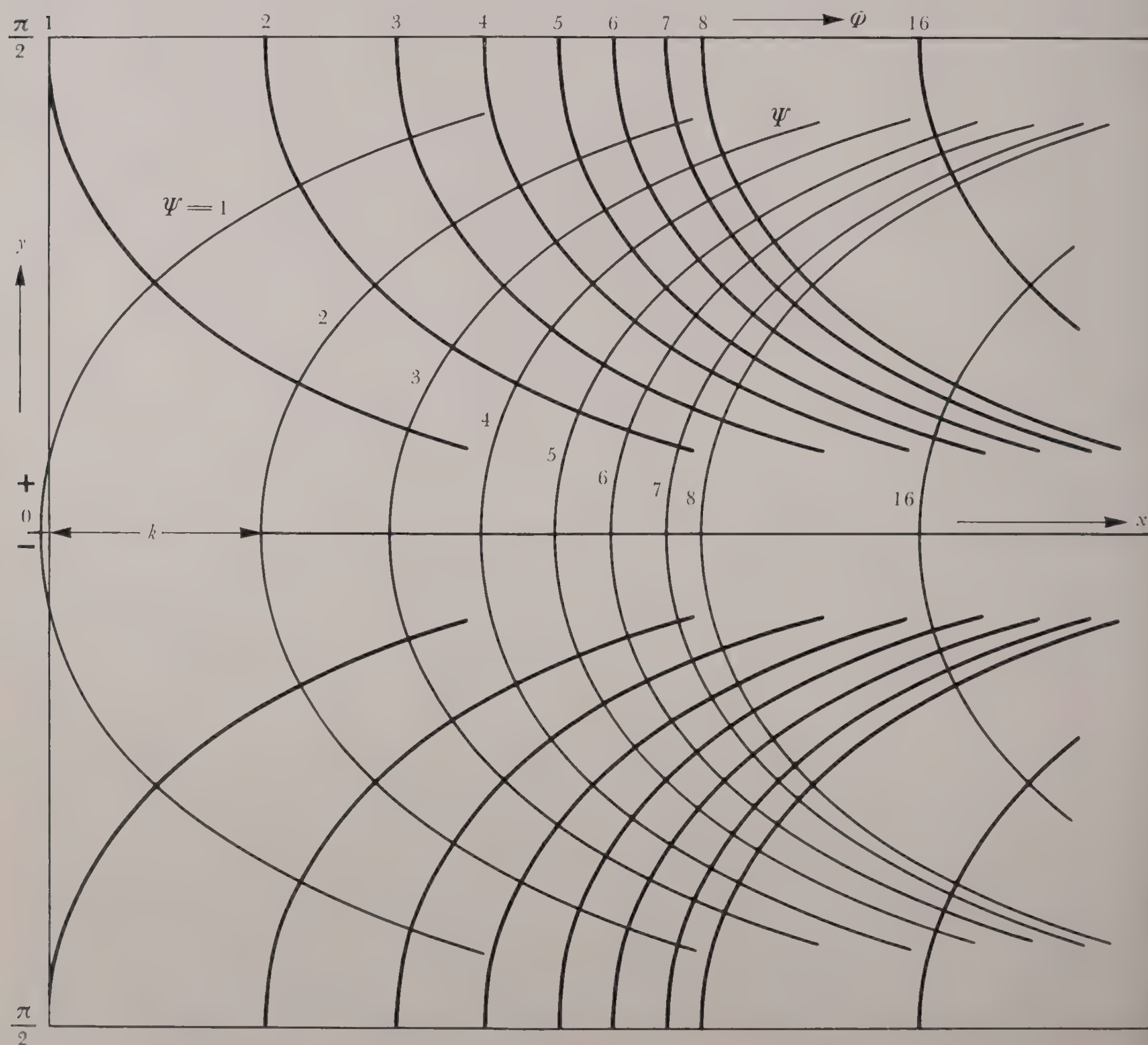

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Fig. 4. – Linear field in a zone with parallel boundaries (parallel strip)

k = Parameter of the curve

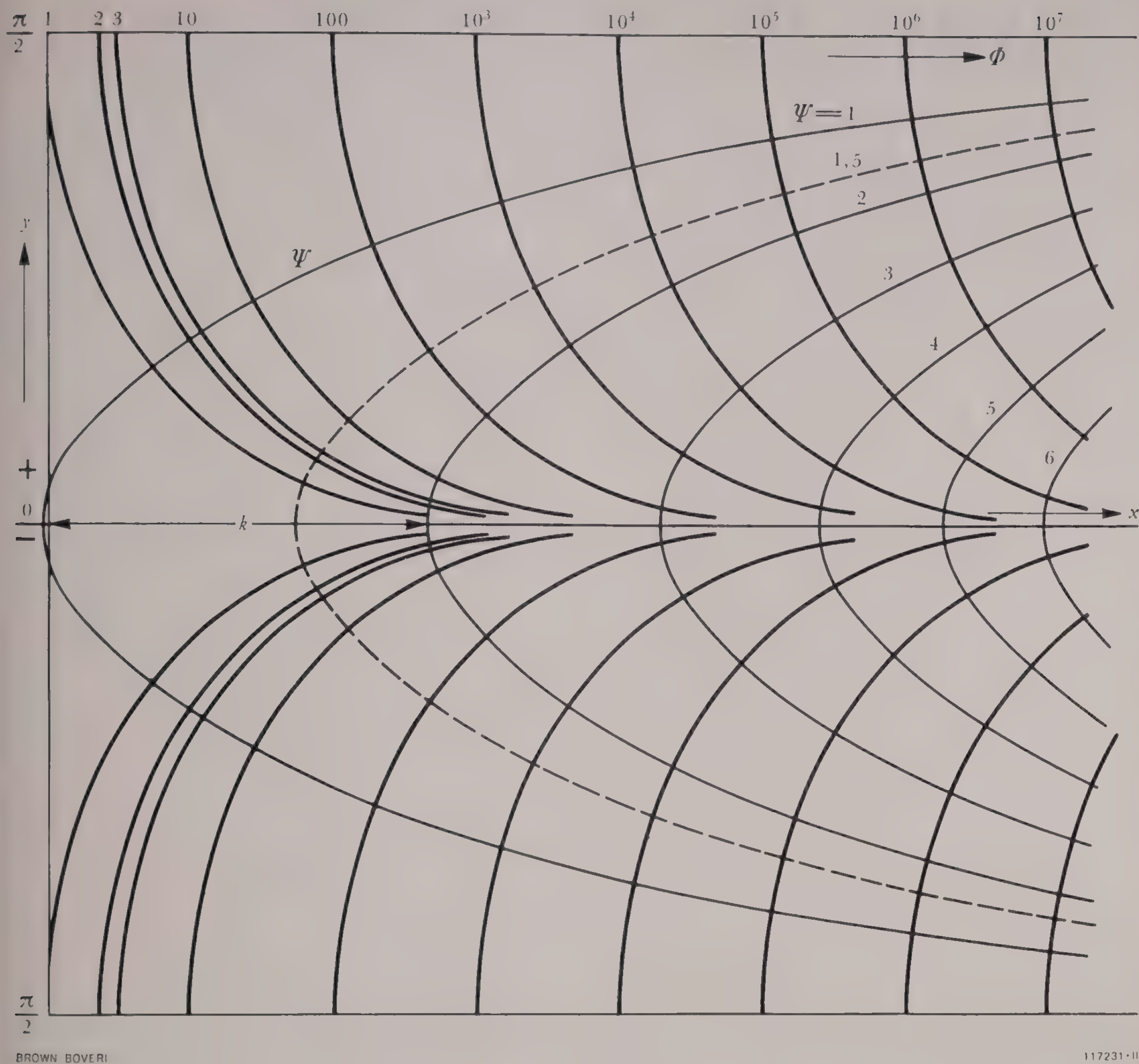

Fig. 5. – Non-linear field in a zone with parallel boundaries

The numbers included indicate the extraordinary increase in the potential Φ with x.

k = Parameter of the curve

When general boundaries are imposed on the extent of the field, the magnetization curve and Maxwell's equations yield non-linear partial differential equations. In certain cases these may be converted to ordinary equations, two of which will now be discussed.

The Field in a Zone with Parallel Boundaries (Parallel Strip)

Let us consider a field which flows in one direction in one-half of a parallel strip, is bent back towards the middle of the strip and flows back in the other half of the strip. In the linear case (Fig. 4) the equation for a field curve may be expressed by $x = k - \log_e \cos y$, where x and y are the co-ordinates along and across the strip, respectively; k is a constant. The width of the strip is made π. If one flux line is displaced parallel to the strip it coincides with another line. It can also be shown that the field strength increases exponentially with the co-ordinate x.

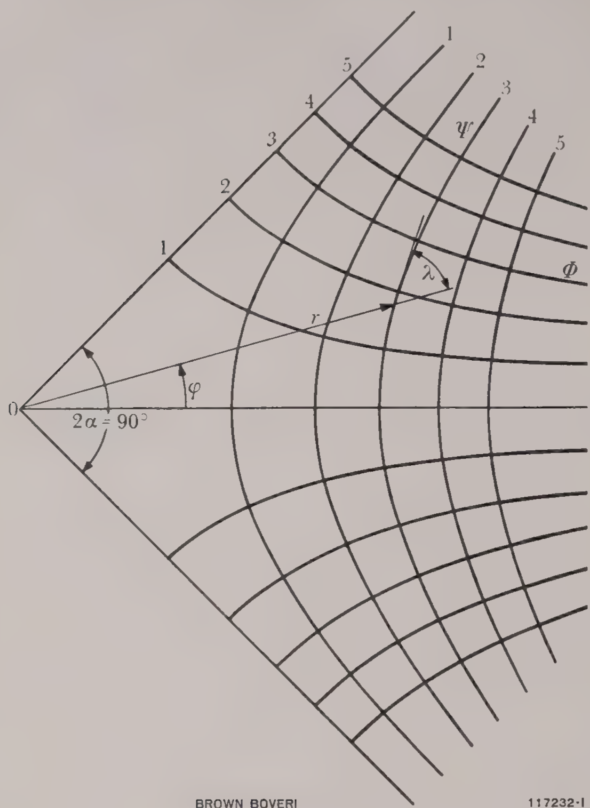

Fig. 6. – Linear field in an angle of 90°

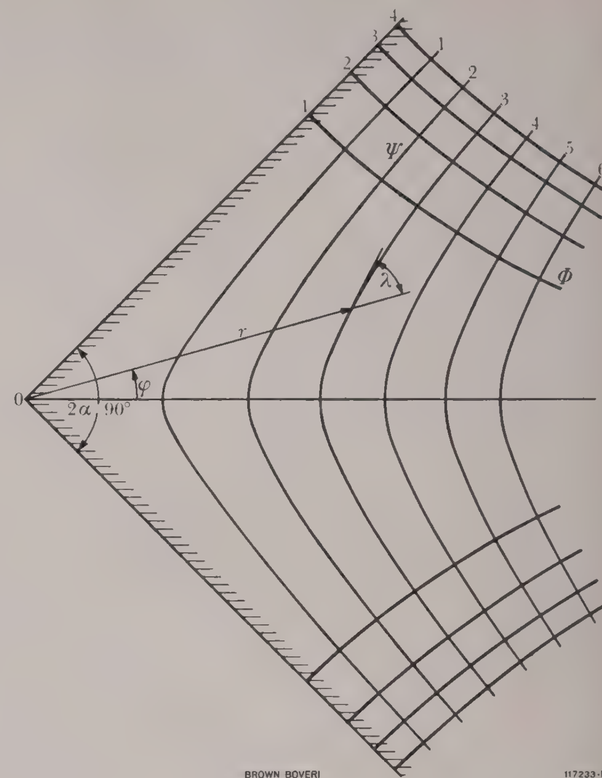

Fig. 7. – Non-linear field in an angle of 90°

The angular zone for a field running in saturated iron is marked by the hatched lines.

If we now postulate the hypothesis that the last-mentioned two properties also apply in the non-linear case, the partial differential equations are reduced to ordinary equations. It is apparent that no contradiction arises, and that the expression is thus justified. If we use p to denote the derivative dx/dy of a field curve, the following parametric representation of a flux line is obtained

$$y = \arctan p - \frac{n-1}{n+1} \frac{p}{1+p^2}$$

$$x = \frac{n}{n+1} \log_e (1+p^2) - \frac{n-1}{n+1} \frac{p^2}{1+p^2} + k$$

The flux and potential lines are plotted in Fig. 5. It will be observed that the flux lines are compressed far less towards the edges of the strip than they are in the linear case. But in the middle of the strip they are bent much more.

The Field in an Angle

The arrangement of the fields investigated in this case can be seen from Fig. 6 to 9. The angle $|\varphi| < \alpha$ may be assumed to represent iron, the remainder air. We are looking for a field which is completely in the iron; hence the magnetic field in the air is ignored. Owing to the high permeability of the iron, except when heavily saturated (above about 18 kGs), this does not cause any serious error.

This case often occurs in practice in the vicinity of edges, e.g. in transformers at the junction between the core and the yoke and at the bends in the yoke, in generators between the teeth and the yoke of the stator laminations, or between the teeth and the rotor body. Particularly frequent are the right-angle bends $2\alpha = 90°$ and $2\alpha = 270°$.

The formula obtained can be utilized to calculate the field of a plane dipole, by making $\alpha = -90°$ (Fig. 10 and 11).

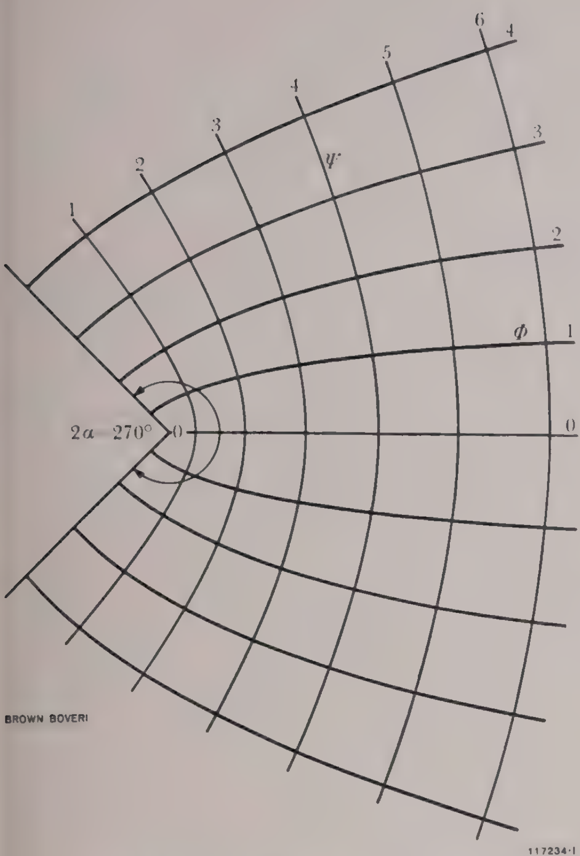

Fig. 8. – Linear field in an angle of 270°

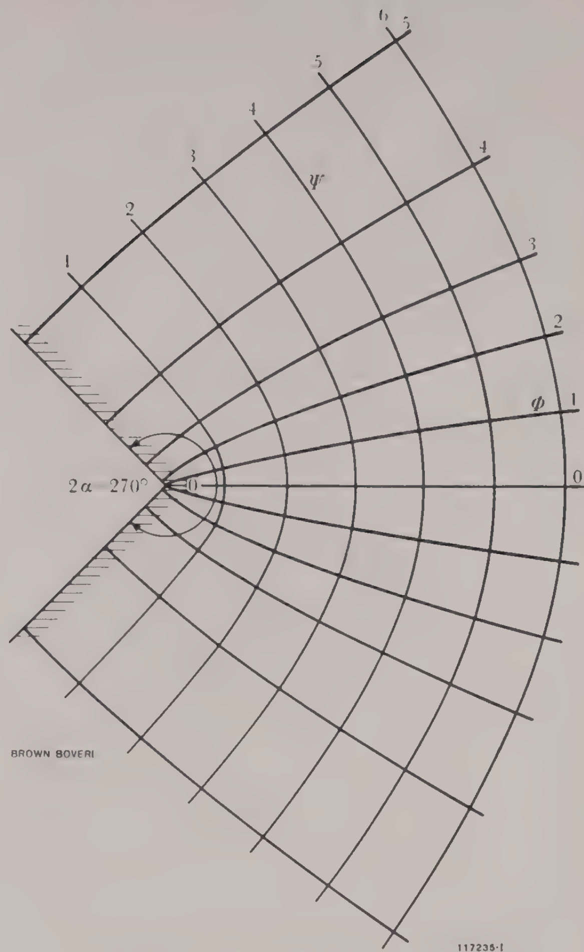

Fig. 9. – Non-linear field in an angle of 270°

The angular zone for a field running in saturated iron is marked by the hatched lines.

In the linear case each of these fields can be considered as one power of a complex variable. In doing so the following properties are observed:

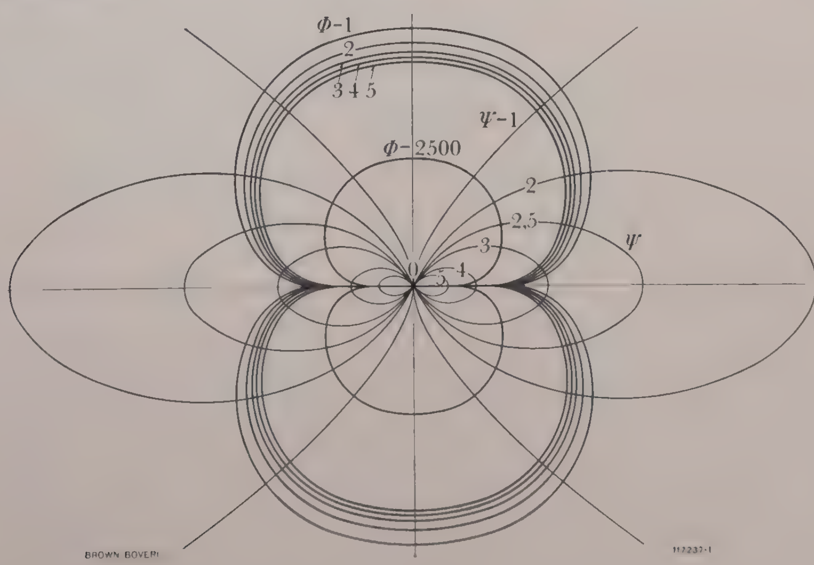

Fig. 10. – Non-linear dipole field

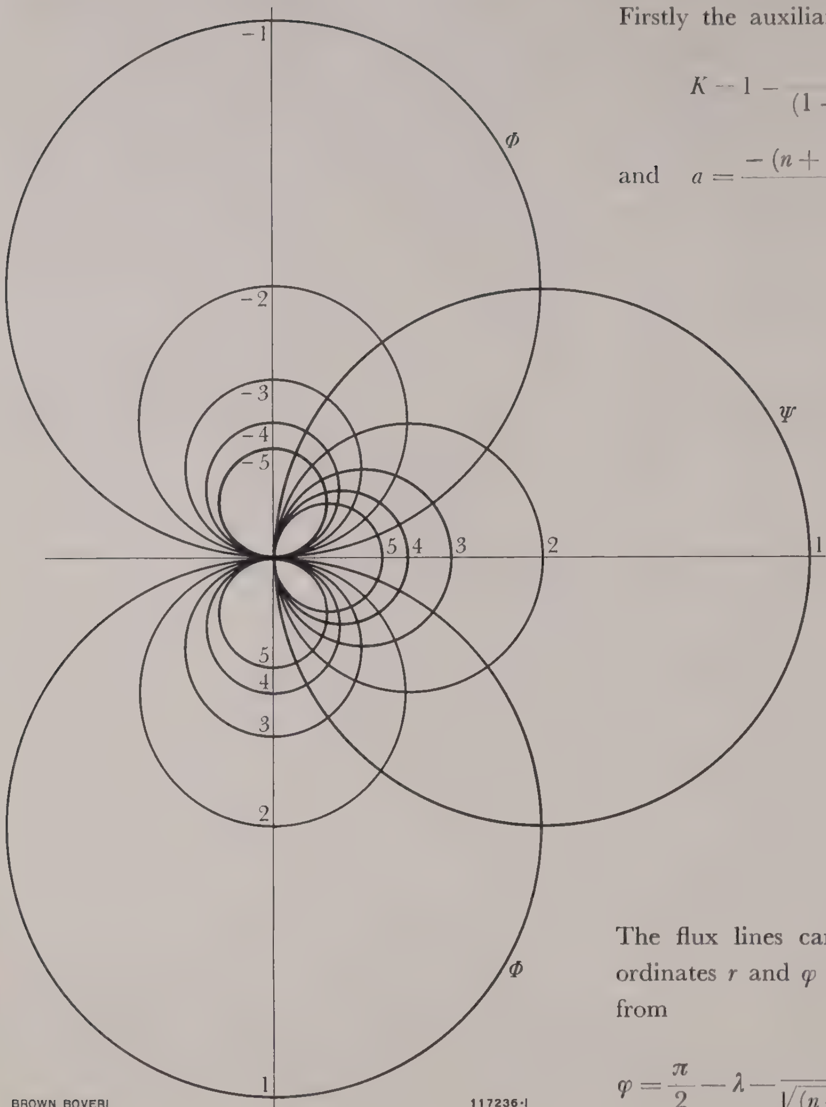

Fig. 11. – Linear dipole field

1. Along each ray passing through the centre 0 (i.e. along $\varphi =$ constant) the field strength increases as a power of the radius r (as, for instance, in Fig. 6).
2. All flux lines are similar.

It can be shown that when the magnetization curve $H = B^n$ chosen is used, both these properties are obtained; also in the non-linear case. With this hypothesis the partial differential equations are reduced to ordinary equations, which can be exactly integrated.

The calculations can thus be reduced to the following final formulae:

Firstly the auxiliary quantity a is calculated from

$$K = 1 - \frac{1}{(1 - 2\alpha/\pi)^2}$$

and $$a = \frac{-(n+1) \pm \sqrt{(n+1)^2 - 4nK}}{2K}$$

The flux lines can now be calculated in polar co-ordinates r and φ with the aid of the parameter λ from

$$\varphi = \frac{\pi}{2} - \lambda - \frac{a}{\sqrt{(n+a)(1+a)}} \operatorname{arc\,tan} \left\{ \sqrt{\frac{1+a}{n+a}} \tan \lambda \right\}$$

$$r = \frac{1}{\sin \lambda} \left(1 + \frac{a+n}{a+1} \cot^2 \lambda \right)^{-\frac{a}{2(n+a)}}$$

The parameter λ denotes the angle between the radius vector and the field line. The non-linear flux and potential lines Ψ and Φ, respectively, are illustrated in Fig. 7, 9 and 10, while Fig. 6, 8 and 11 show the corresponding linear fields. Here too it may be seen that the flux lines are less severely compressed in the non-linear case.

Although the selection of calculated non-linear field diagrams is still very modest, compared with the linear cases, those reproduced in this article nevertheless indicate that it is not always impossible to calculate them.

(KME) A. DALCHER

THE ANALOGUE COMPUTER, ITS EMPLOYMENT AND IMPORTANCE IN THE COMPUTER CENTRE

681.14:530.17

The present article deals with the principle, operation, application and importance of the analogue computer installed in the Brown Boveri Computer Centre. This machine is used for a wide variety of tasks, frequently acting as a simulator with normal equipment. With it the practical effect of any improvement made in the overall system can be observed directly. The article concludes with an example from the field of process control, pointing out the importance of the analogue computer as a means of predicting the operational behaviour.

THE COMPANY has now possessed an analogue computer for about four years. This review of the experience gained in that period will give some idea of its application and importance.

Principle of the Analogue Computer

By way of introduction, the basic nature of the analogue computer may be briefly outlined, although there are plenty of references in the literature on this subject. It includes elements for all the elementary mathematical operations, such as algebraic summation, multiplication, integration and the evaluation of non-linear functions. These operations can be represented with great accuracy in a frequency band between zero and about 50 c/s.

The solution of a problem with the aid of the analogue computer may be divided into a number of phases, the first of which is an analytical phase—the installation or machine to be investigated being split into its elementary parts, although it must be added that such investigations are only undertaken for concrete objects. This analysis then provides a group of mathematical equations or, in the particular case of a control problem, a block diagram containing all the important functions. Moreover, the analysis provides a closer insight into the nature of the system being investigated and, by omitting minor details, allows the principal functions to be recognized.

The second phase is the synthesis in which, with the aid of the various elements of the analogue computer and according to the system of equations previously determined, or the selected block diagram, the system is simulated, or a model analogue to the system under investigation is set up. From the accuracy with which the model is able to reproduce certain known functions of the object, it is possible to confirm the justification for the approximations chosen in the analytical phase.

The next phase is the operation of the system, as simulated on the computer. The freely chosen parameters may be varied, readings are taken and any improvements in the general arrangement which appear necessary can be made. The results so gained are then communicated to the interested departments.

For all applications in the sphere of control engineering involving high accuracy, computing elements are required which possess still higher accuracy and stability than the elements being investigated. To cope with the steadily advancing perfection of engineering techniques, the analogue computer has also to be continuously extended, with the advantage that its capacity is increased at the same time. Practical experience shows that analogue computers can be employed to solve problems of increasing importance, necessitating the employment of a growing number of elements.

Importance of the Analogue Computer in an Industrial Undertaking

The foregoing remarks dealt with the phases preceding the solution of a problem; these already provide quite a close insight into the inner structure of the task on hand. As soon as the circuitry and arrangement have been settled, the observation of the mutual effect of the various variables provides extremely valuable information, particularly as it is an easy matter to take readings simultaneously on several different elements. The effect of every improvement made to the arrangement can be checked immediately.

In certain cases the circuitry on the computer can be set up to correspond exactly to the block diagram of the installation being investigated, without having to resort to mathematical methods. The skilled operator can set up the analogue diagram with very little extra effort, the great advantage of which is that the circuit is right before his eyes and he can correct any errors at once.

Mathematical Theory and the Analogue Computer

As far as the comparison between mathematical theory and analogue calculation is concerned, it may be established that the analogue computer is primarily an experimental device whose accuracy often has to be checked by mathematical methods. The computer may also be able to augment the theoretical approach to a solution, in that by varying the variable parameters it provides indications of where the theoretical treatment has to commence.

Analogue Computers as a Means of Instruction

The ability to couple an analogue computer with normal equipment has already been mentioned. With this interesting combination it is possible to study the overall system under circumstances closely resembling those encountered in actual service. This leads to a further application of the analogue computer, namely as a means of instructing operating staff—a method which has been standard practice for some time in air transport undertakings, for example—especially when they have to control equipment which can be partly or wholly simulated on the computer. Particularly fruitful results have been obtained, for instance, in training staff concerned with the commissioning and operation of control systems, where it is necessary to demonstrate the manner in which an optimum control can be exerted, in the shortest possible time, without the installation suffering any ill-effects. For modern control systems with an increasing number of stipulations, this method of instruction continues to gain in importance, especially as regards the time saved in searching for the optimum conditions.

Operation of the Analogue Computer

To judge the practical usefulness of the analogue computer the following aspects must be considered. As already mentioned, the computer is primarily an item of experimental equipment. Generally speaking, it is not suitable for routine work because the arrangement of its elements is usually complicated and is subject to certain contingencies, even when operated by someone having perfect knowledge of the installation. A certain time always has to be spent before it can be actually put into operation, owing to the number of checks which have to be performed. Since the structure of an analogue computer is very complex and sensitive, it requires a specialist to operate it. This operator then has the task of executing the more difficult problems and lending his assistance to other users of the computer. In this capacity he must be able to grasp a wide variety of technical problems in a very short time, and to analyse them, for which he must possess considerable mathematical ability. To enable him to deal with the often complex problems, he must also be able to think on synthetic lines.

It should be most instructive for this specialist to be allowed to take part in the commissioning of equipment which he has previously investigated on the computer. It also enables him to widen his experience of the reciprocal effects of the various quantities involved.

Typical Application

An example of the application of the computer is the investigation of a system for process control. For problems of this kind it is impossible to check the behaviour of the complete system in the laboratories of the firm supplying only separate elements of the system, because the controlled object, the main part of the whole system, is not available. With the analogue computer it is possible in most cases to simulate this object fairly closely, so that the dynamic response of the whole control system can be predicted with sufficient accuracy.

In Fig. 1 the servomotor acts as the correcting unit and, via valve 2, varies the rate of flow. The servomotor is controlled by the dashpot regulator (amplifier) 3. These parts are mounted on a test-bench in the workshop. The other parts of the control system, i.e. the electronic amplifier 4 and the analogue computer 5 are situated in the Computer Centre about 100 m away from the test bench. As indicated in the sketch, a multicore cable acts as the connecting link between the two parts of the control system.

In the computer the amplifier A_1 and the connected resistors and capacitance simulate the controlled object, which in this case can be represented by a single time constant. A_2 and A_3 represent the transfer functions of the measuring system, which can be described by two time constants. A_5 produces the reference input depicted in Fig. 2, while A_4 acts as summation element. A_6 merely serves a purpose

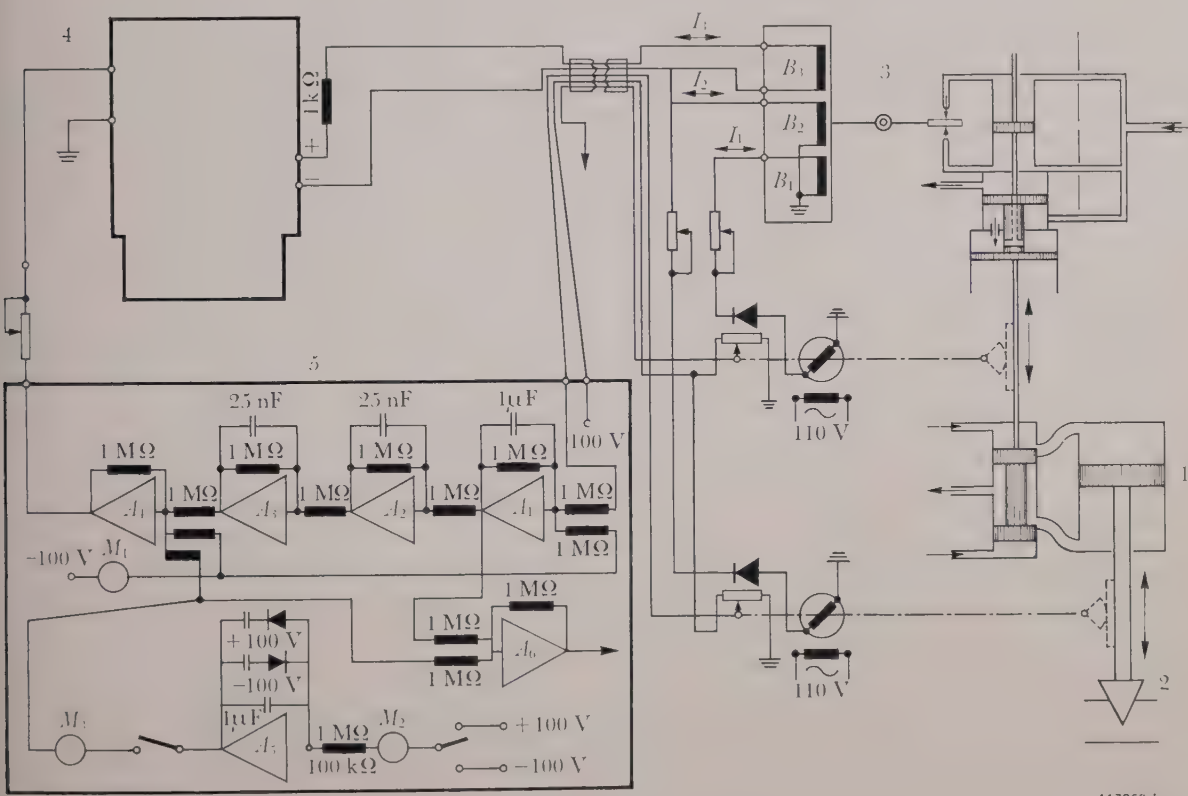

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Fig. 1. – The control circuit being investigated

1 = Servomotor
2 = Valve
3 = Dashpot regulator (amplifier)
4 = Electronic controller
5 = Analogue computer
A_1–A_6 = Amplifiers

M_1–M_3 = Setting potentiometers
I_1–I_3 = Control currents of 3
I_1 communicates the feedback of the control valve of 1
I_2 communicates the feedback of valve 2
B_1–B_3 = Control solenoids

The components 1 and 3 are situated in a part of the factory, while 4 and 5 are in the Computer Centre. The signals are transmitted over a multicore cable.

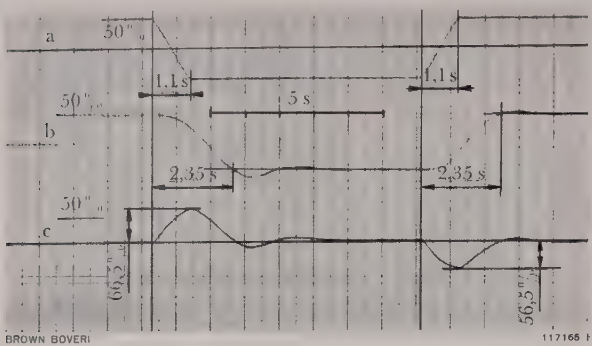


Fig. 2. – Traces obtained during an extensive series of tests

a: Desired value
b: Actual value
c: Difference between a and b

associated with the instrumentation and produces the difference between the reference value and the actual value.

The upper part of Fig. 2 shows the master or desired value, the middle section the actual value, and the lower section the difference between the two. It will be observed that the actual value responds to changes in the desired value with a considerable time-lag, but that there is very little overswing.

As a result of this investigation it was possible to predict that the components of the control system to be delivered would exhibit the correct response.

(KME) P. COROLLER

METHODS OF INVESTIGATING ELECTRICITY NETWORKS

621.316.313

After briefly mentioning two mathematical methods, the author describes the investigation of electricity systems with the a.c. network analyser, the remarks being illustrated by reference to practical examples. Some useful tips are given regarding simulation techniques. These are followed by reference to some more recent methods, e.g. coupling the network analyser to the analogue computer, and digital computation of the network by means of the program-controlled computer.

On the Principles of Investigating Electricity Networks

ANYONE confronted with the task of calculating the conditions in an electricity network will immediately recognize the characteristic difficulty in the handling of such problems: Having regard to the particular configuration, every network requires a fresh calculation, starting from scratch, involving an extremely large number of variable quantities.

For the easier investigations two possible methods immediately spring to mind. The first method is to simplify the configuration of the network by eliminating the junction points. The original network is replaced by an equivalent network, enabling the problem to be solved more easily. It is also possible to carry out a reverse transformation, thus determining the voltages and currents in the original network step by step [1].

The second method is to set up the linear system of equations from which the currents or voltages of the various links forming the network can be determined [2].

However, for the following reasons it is impossible to employ such a method when calculating by hand or with an ordinary office calculating machine, except perhaps in certain exceptional cases.

The given quantities are usually not the resistances, voltages or currents, but mixtures of resistance values, absolute values of voltages and some of the powers. The unknowns are then the remaining voltages and powers.

Furthermore, as regards the electricity networks, the following development may be observed:

- The networks are continually growing;
- Interconnection is increasing;
- The number of points where junctions are made with neighbouring networks is growing;
- The number of infeed points is increasing, either from the network's own generating stations or from neighbouring networks;
- The number of consumers or shunt links in the interconnected system of networks is always being enlarged. With the use of progressively higher voltages, the importance of capacitive shunt branches grows too, with the result that the number of links in the system continues to increase.

Hence, as a rule, it is only possible to investigate electricity systems with the aid of analysers or digital computers.

Analogue Devices

The d.c. network analyser, the analogue computer [3] and the a.c. network analyser [4] are all analogue devices. The first two operate in principle with direct current, the third with alternating current. W. J. Karplus [5] places the network analysers in the category of direct analogue devices, but the analogue computer in the indirect category. This classification expresses the main difference in their operating principle.

Direct Analogy

In a direct analogy one and only one physical quantity in the simulation corresponds to one physical quantity in the original. Thus, for instance, in

the thermo-electric analogy, temperature difference corresponds to voltage difference, thermal resistance to ohmic resistance and the flow of heat to current. When determining the natural frequency of a shaft subjected to pulsating torques, the moment of inertia corresponds to the inductance, the angle to the current, and the reciprocal elastic torque to the capacitance. If a dual image of an electricity system is set up, a current source corresponds to a voltage source, an inductance to a capacitance and vice versa. The particularly lucid case, in which corresponding values are represented by the same physical quantities in both original and image, are contained as a special case in this direct analogy.

Indirect Analogy

In contrast, in the indirect analogy, different quantities in the original are represented by a single physical quantity in the set-up. Thus, for instance, when setting up an electricity system on the analogue computer, the current and its integral, the quantity of electricity, are determined by measuring a voltage.

The next chapter will describe the investigation of electricity networks, using an a.c. network analyser.

The A.C. Network Analyser

The Brown Boveri a.c. network analyser (see Fig. 3 on page 285) contains several voltage sources, auto-transformers and variable impedances, which are used to simulate the generators, transformers, transmission lines, shunt reactors, synchronous capacitors, capacitances and loads in the network being investigated. It is equipped with metered and unmetered jumpers, as well as shorting links. The available elements are listed in the table of active and passive elements on page 315 opposite.

Of the twelve generators, five exhibit a reactance in the range from 0 to 6·21 per unit, ten generator elements can be loaded up to 10 per unit and two up to 20 per unit. Six capacitive elements can be set at up to 7·21 per unit. Other elements which can be plugged in at the rear of the analyser allow the maximum value to be increased to 11·21 p.u. A number of the auto-transformers allocated to the load circuits can be varied down to 0·4 p.u. Furthermore, these auto-transformers and those in the succeeding lines can have their voltages varied in steps of 0·005 p.u. The number of insulating transformers has increased to ten.

Having made all the necessary settings, and on completion of balancing, minor changes in the circuitry (e.g. opening lines, switching on loads, causing short circuits, etc.) and any adjustments to the amplitude and phase angle of the voltage sources can be made from the control desk. Special measuring amplifiers in the generator elements and in the metering desk prevent the instruments from affecting the image of the network. All passive elements can be adjusted in decades. As a direct analogy of the original network, the analyser is an extremely adaptable device and provides a very clear picture of the network being investigated.

Reference to Completed Investigations

As examples of the numerous investigations which have been carried out with the a.c. network analyser, the following are worth special mention.

Flow of power

Interconnection of the networks of a number of electricity undertakings under normal operating conditions and with special connections. The investigation of successive stages in the expansion of networks at home and abroad, including railway traction networks, urban supply systems which may already be in existence, or due for reorganization or extension, and finally factory supply systems.

Faults

Included under this heading are investigations to determine the short-circuit powers in the event of three-phase short circuits (e.g. to determine the circuit-breaker rating needed). The a.c. network analyser can be used to represent networks with negative reactances (three-winding transformers) and branches of any desired impedance angle. The problem which had to be investigated most often was the single-pole earth fault in a network with earthed neutral. Other investigations were concerned with the line-to-line fault, single-phase open circuits, asym-

Active and passive elements of the a.c. network analyser

No.	Designation	Total quantity	Of which	Value to be set	Range (in the per unit system)	Step (in the per unit system)	Remarks
1	Generator element	12		Voltage U Angle φ	0–2·5 0–360°	continuous	Output terminals shorted when "Off"
2	Generator reactance	12	7	Reactance ωL	0–1·220	0·001	—
			5		0–6·21	0·001	from 0–1·220
						0·01	from 1·00–6·21
3	Line element	114	90	Resistance R and	0–1·220	0·001	—
4			24	reactance ωL	0–6·21	0·001	from 0–1·220
						0·01	from 1·00–6·21
5	Capacitive element	60	12 42	Susceptance Y	0–0·621 0–1·221	0·001	—
6			6		0–7·21	0·002	from 0–1·220
						0·01	from 1·00–7·21
7	Load element	27		Resistance R and reactance ωL	0·2–16·2	0·01	Connection in series or parallel
8	Auto-transformer for load element	27	12	Transformation ratio	0·7–1·31	0·01	with 2 "Off" positions
			15		0·4–1·315	0·005	
9	Auto-transformer	12		Transformation ratio	0·7–1·315	0·005	—
10	Insulating transformer	10	6 2 2	—	—	—	Ratio 1:1 $1:\sqrt{2}$ $1:\sqrt{3}$
11	Unmetered jumper	48		—	—	—	non-measurable
12	Metered jumper	24		—	—	—	measurable

For use in the analyser the per unit system is based on the following rules: The rated value (15 V) of the output voltage of the generator element is denoted as 1, the rated value of the current (30 mA) also as 1. All passive elements contained in the analyser are appropriately calibrated according to the per unit system and marked. The unit of resistance is 15/0·03 = 500 ohms and that of the conductance 0·03/15 = 0·02 mho.

metric branches connected in series (a capacitor short-circuited in one phase) and faults in star/delta transformers. These investigations are carried out with the methods employing symmetrical components [6] or α, β, 0 components [7]. When the symmetrical components are employed, the generator reactances in the positive, negative and zero-sequence systems can be simulated very exactly. As a normal rule, the

negative and zero-sequence systems only contain passive elements and can therefore be replaced by pre-determined branches or polygonal networks, depending on the case in point. In contrast, active elements have to be inserted in both the α and the β system. However, the α, β, 0 components, compared with the symmetrical components, offer the advantage that, for the equivalent circuits for any kind of multiple fault, only insulating transformers are required and no means of rotating the voltage and current vectors [8]. A further problem belonging to this category was that of resonance with harmonics of the 5th, 7th, 11th and 13th order. Long lines in the original network were represented by π lines. The network was simulated for the 6th and 12th harmonics and the frequency varied continuously between the 5th and 7th, or between the 11th and 13th order.

Stability

Particularly interesting problems are encountered in stability investigations. The step-by-step method demands great concentration, but it underlines the special advantages of the analogue method.

By continuously determining and adding up the angular differences the engineer is simultaneously able to realize why individual machines and the whole system behave the way they do and not in some other manner. He gains insight into the effect of the various parameters and, by different measures, is able to control the process so that the desired effect is achieved.

The problems dealt with have concerned the stability of networks with synchronous generators in hydro-electric, steam and diesel-powered generating stations, with synchronous motors, synchronous capacitors and frequency converters; also with symmetric and asymmetric faults with progressive adjustment of the loads, as well as cases which could only be simulated artificially (dummy loads on the generators).

Special problems

These include numerous transfigurations of networks into branches or equivalent triangles or polygons; also compensation circuits, dimensioning the means of modulating for variable frequency, measuring equipment for unbalanced loading, circuits for converting the component values into phase values (symmetric components) and natural frequencies of shafts subjected to pulsating torques.

The smallest network investigated by the analyser contained 5 branches, the largest over 200. The statistical break-down of the network investigations is as follows:

57% for the determination of power flow, and the distribution of voltage and current,

25% for the investigation of symmetric or asymmetric faults,

10% for investigations of steady-state and transient stability,

8% for electrical and mechanical problems of a special nature.

Special Artifices

Three-winding transformers

These can be simulated by a reactance star [9], in which it is by no means rare for negative reactances to occur. A small negative reactance corresponds to a large capacitance, which sometimes exceeds the maximum value available on the analyser.

By utilizing a star/delta transformation a reactance star can be converted into an equivalent delta. If $\bar{Y}_{S1}$, $\bar{Y}_{S2}$, $\bar{Y}_{S3}$ denote the susceptances of the limbs of the star, and if the subscript 3 is allotted to the limb of the triangle across limbs 1 and 2 of the star, its susceptance $\bar{Y}_{D3}$ is given by

$$Y_{D3} = \frac{\bar{Y}_{S1} \cdot \bar{Y}_{S2}}{Y_{S1} + Y_{S2} + \bar{Y}_{S3}}$$

Multiplying both sides of the above equation by $\bar{Y}_{S3}$ yields

$$\bar{Y}_{S3} \cdot \bar{Y}_{D3} = \frac{\bar{Y}_{S1} \cdot \bar{Y}_{S2} \cdot \bar{Y}_{S3}}{\bar{Y}_{S1} + \bar{Y}_{S2} + \bar{Y}_{S3}}$$

Thus if the susceptance $\bar{Y}_{S3}$ of one limb of the star is large, from the absolute aspect, that of the opposite side of the triangle $\bar{Y}_{D3}$ will be correspondingly small, i.e. more favourable for the analyser. In that case it is preferable to replace the reactance star with a negative reactance by the equivalent delta.

Transformers with combined direct and quadrature regulation

In this form of regulation the direct-axis components are obtained by proportionately raising or lowering the transformation ratio by means of an auto-transformer. The quadrature-axis components can be realized by simulating the whole system twice, once in an α system and once in a β system [10]. With an angular displacement of, say 60°, the ratio of the direct to the quadrature-axis component is $1:\sqrt{3}$. The components are therefore conveniently composed with insulating transformers having the ratio $1:\sqrt{3}$.

Leg of a polygon with negative resistance [1]

The impedances of individual legs of a polygon equivalent to a network sometimes contain real components with a negative sign. This state of affairs occasionally occurs during the transfiguration into an equivalent delta of several consecutive line sections with infeed reactances. If no special elements are available for simulating the negative resistances, it is possible to circumvent the negative resistance of the leg of the polygon by using passive elements.

In the triangle (Fig. 1) R_1, X_1 and R are all positive values. The leg AB has an impedance with a negative real part, while AC and BC have the same ohmic resistance. This delta can be converted into a star, whose impedance only contains positive real parts, provided the inequality

$$\left(\frac{R_1}{2}\right) < R < \frac{R_1}{2}\left[1+\left(\frac{X_1}{R_1}\right)^2\right] \qquad (1)$$

is fulfilled.

Any side of an n-sided polygon forms triangles with the $(n-2)$ points not on the side in question. If one of these sides has an impedance with a negative real part, of the form $Z = -R_1 + jX_1$, first a resistance R is chosen, which fulfils the above inequality (1). Then from the $(n-2)$ triangles a suitable triangle is sought, whose legs connecting to an opposite point of the polygon fulfil the following condition: The real parts of the conductances of the two legs must not be less than the reciprocal of R. The resistances R can now be split off these two legs in the form of parallel resistances. These, together with the leg having the impedance $\bar{Z}_1$ form a triangle which can be transformed into a star having no negative resistances, so that now all legs of the new equivalent part of the network can be simulated by passive elements.

By way of example, let us assume a triangle whose legs have the following impedances, as shown in Fig. 2a:

$$\bar{Z}_1 = -R_1 + jX_1 = -0{\cdot}1 + j3{\cdot}6$$
$$\bar{Z}_2 = 0{\cdot}06 + j0{\cdot}32$$
$$\bar{Z}_3 = 0{\cdot}08 + j2{\cdot}1$$

To determine the resistances R belonging to the first leg AB, the inequality (1) must first be set up. With $(R_1/2) = 0{\cdot}05$ and $(X_1/R_1) = 36$ it reads

$$0{\cdot}05 < R < 64{\cdot}85$$

and correspondingly, for the conductance G

$$0{\cdot}01542 < G < 20 \qquad (2)$$

The susceptances of the legs AC and BC work out to $0{\cdot}56604 - j3{\cdot}01887$ and $0{\cdot}018114 - j0{\cdot}47550$, respectively. Their two real parts are larger than the lower limit in (2) so that the negative resistance of the first leg can be eliminated. If G is made $0{\cdot}018114$, the legs AC and BC can be resolved into two components each, namely $\bar{Z}_2'$, R_2 and Z_3', R_3, as shown in Fig. 2b. The dotted triangle consisting of the leg with the negative impedance Z_1 and the two ohmic resistances $R_2 = R_3$ is now transformed into the equivalent star. In this way Fig. 2a is converted into the network in Fig. 2c containing no negative resistance.

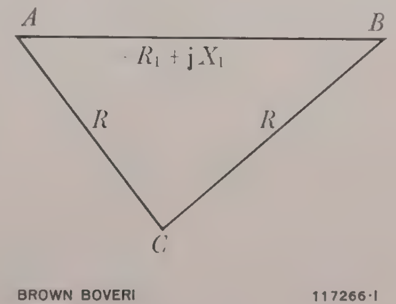
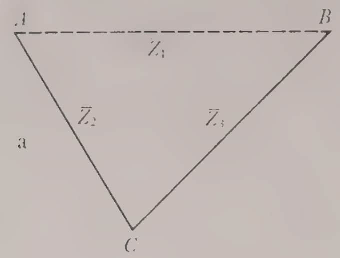


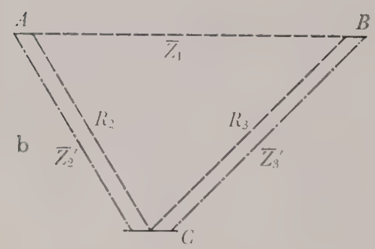
Fig. 1. – Delta network

The leg AB contains a negative resistance. The other legs have equal ohmic resistances.

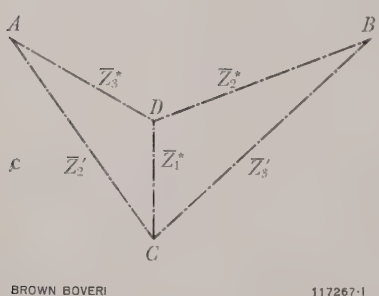
[1] We are indebted to W. F. GRAHAM, Bureau of Reclamations, Denver, USA, for bringing this method to our attention.



Imp.	p. u.
$\overline{Z}_1$	$-0{\cdot}1+j3{\cdot}6$
$\overline{Z}_2$	$0{\cdot}06+j0{\cdot}32$
$\overline{Z}_3$	$0{\cdot}08+j2{\cdot}1$



Imp.	p. u.
$\overline{Z}_2'$	$0{\cdot}058205+$ $+j0{\cdot}32068$
R_2	$55{\cdot}206$
$\overline{Z}_3'$	$j2{\cdot}1030$
R_3	$55{\cdot}206$



Imp.	p. u.
$\overline{Z}_1^*$	$27{\cdot}599-$ $-j0{\cdot}90068$
$\overline{Z}_2^*$	$0{\cdot}00874+$ $+j1{\cdot}8014$
$\overline{Z}_3^*$	$0{\cdot}00874+$ $+j1{\cdot}8014$
$\overline{Z}_2'$	$0{\cdot}058205+$ $+j0{\cdot}32068$
$\overline{Z}_3'$	$j2{\cdot}1030$

Fig. 2. – Determination of an equivalent network containing no negative resistances

Values expressed in the per unit system

a: Original network. Leg *AB* contains a negative resistance

b: Resolving the legs *AC* and *BC*

c: The resultant equivalent network obtained by the delta-star transformation of the dotted triangle

A.C. Network Analyser in Conjunction with the Analogue Computer

The analogue computer is now accommodated in the room next to the network analyser. Preparations are at present being made for coupling these two machines together. When the analyser operates alone, the amplifiers representing generator elements are fed from the *RC* oscillator, the amplitude and phase angle of the voltage being adjusted at the source or from the desk. When the two machines operate jointly, these settings on the desk are blocked. The voltages required are now fed to the individual amplifiers of the analogue computer via special components, their amplitude and phase angle being set on the analogue computer.

The combination of the network analyser and the analogue computer opens up interesting possibilities for the investigation of electricity networks or other systems. The computer now has a network available, so to speak. The analyser, which was mainly used for steady-state tasks, can now be augmented by automatically controlled machines. Thus, in the near future the following problems can be solved:

- Problems associated with automatic control, including the repercussions from the network;
- The combined effect of control processes considering several machines connected to the same network;
- The effect of switching large machines on to a network;
- The effect of the control system on the magnitude of the short-circuit currents;
- The curve of the short-circuit currents with respect to time;
- Investigation of the transient stability of a number of machines, the oscillation curves being plotted automatically;
- The effect of voltage control on the transient stability;
- Repercussions of its control system on the prime mover.

Digital Methods

The solution of the problems mentioned in the opening chapter already involved digital methods. Although quite neat methods have been devised for network problems solved by means of desk-type calculating machines [2], their use is usually confined to very small networks. When a program-controlled digital computer is employed, digital methods are also suitable for large networks. Thus a new, very versatile instrument becomes available for the investigation of electricity networks.

This computer is ideal for calculations which recur cyclically with a certain regularity. In this way, for instance, shunt branches of consecutive line sections can be eliminated by the star-delta conversion of a group at a time, the network being then systematically investigated with the aid of matrices [11]. The following method, of which only a brief description is given, is useful for investigating the flow of power and stability; this is the iteration method utilizing the equations of the junctions [12, 13].

Power Flow

Starting with the estimation of the voltages, the currents flowing into each leg of the network from a junction point are calculated and added together. The state at this point may be either of two possibilities:

1. The junction is unloaded, so that the sum of the line and capacitance currents must be zero. If this condition is not fulfilled, the voltage has to be changed. The condition of zero current provides two equations from which the supplementary voltage can be determined.
2. The particular junction is connected to a generator or load. In this case there are two more alternatives:
   - The incoming (or outgoing) active and/or reactive power is known, or
   - The active power and the absolute value of the voltage are given.

Here, as with the unloaded junction, two equations can be established, enabling the conditions to be fulfilled when the voltage at the junction has been corrected.

Each junction in the network is then dealt with in this manner, the voltage being successively corrected and this correction stored if it exceeds the correction of the preceding point. Having dealt with all junctions, the next question to be settled is whether the maximum correction is smaller than a certain limit. If this is so, a stop command is issued; if not, a new cycle commences, the procedure being continued until the correction is sufficiently small. Following the calculation of the voltage, the flow of power is determined. The result appears in the form of a printed table, the amplitude and phase angle of the voltage appearing as a per unit value for each junction, while the powers of the feeding generators and the loads, as well as the powers flowing towards other junctions, are expressed in MW and MVar. In addition, the power losses in the network can also be printed out.

Stability

To determine the oscillation curve of synchronous machines it is necessary to know the amplitude and phase angle of their e.m.f. in the steady state, i.e. before the occurrence of a fault. Following the determination of the generator outputs, by the analogue or digital method, these e.m.f. are easily determined, provided the transient reactances are known. The solution is then obtained by integration of the differential equations of the synchronous machines, the active powers of the generators being determined each time by transferring to a power-flow program. This procedure may at first appear rather clumsy. At the beginning of the initial time phases several cycles are necessary owing to the changes in voltage. But subsequently the voltage differences are quite small in view of the short time interval. The junction voltages determined at the beginning of an interval provide a good approximation for the subsequent time value, so that, usually, only a small number of cycles are required. The line affected by the fault can be disconnected or reconnected, as desired.

This method, however, offers the advantage that the network is available in its original state and not in a simulated state. This makes it possible for any supplementary values which may perhaps be required to be determined, such as the voltages at individual junctions or the currents in circuit-breakers.

The tape on which the results are printed out gives the angles of the e.m.f. and their derivatives relative to any chosen machine, or with reference to a fixed voltage (infinite supply assumed).

Analogue and Digital Methods

Both methods—the analogue and the digital—exhibit some striking advantages as regards their application for the investigation of networks.

With the analogue method every parameter, i.e. every load, every line, every transformer and every generating station is available for carrying out a particular adjustment. If the results of a simulation are presented on paper, questions such as the following may arise:

What happens when

- this transformer capacity is doubled?
- this line is only run as a single circuit?
- these two sets of busbar systems are operated separately?

By making the appropriate settings on the elements (3–6 selector switches have to be operated on the analyser) or by forming two junctions, a result is obtained which shows whether this change serves a useful purpose, or whether its effect is so small as to be insignificant. Especially during investigations connected with the extension of networks in stages, the analyser is an ideal aid to the supervising engineer as it allows him to look for the most favourable solution. He knows exactly what means he can call on in the particular year and he can utilize these in the most suitable combination. All settings on the active and passive elements, as well as the circuit, can be quickly and reliably checked. The measured power values are entered in on the network diagram direct and immediately checked by adding round the junction. This representation gives a very clear picture of the flow of power in the network.

The digital method allows large, completely meshed networks with numerous feed points and loads to be investigated. A further advantage is the higher accuracy attainable with this method, although this causes a corresponding increase in the time taken (more cycles are needed). It is thus easier to compare different variants with one another as regards their respective losses.

A feature of the digital method is that it allows the methods used previously to be refined by omitting simplifying assumptions, and the process to be spread over a longer period. Furthermore it will allow problems to be investigated which presented insurmountable difficulties previously, either owing to the time taken or the size of the network in question. Compared with the analogue method, the digital method appears to be most suitable for those networks for which a preset plan can be given, according to which the investigation has to be carried out. Having regard to the wide variety of tasks, which the arrangement, operation and extension of electricity networks impose, it is advantageous to be able to adopt the one approach or the other, with the aid of the analogue or the digital methods.

(KME) E. Fässler

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THE USE OF DIGITAL COMPUTERS TO EVALUATE MEASUREMENTS ON THERMAL TURBO-MACHINES

681.14:621-135

To evaluate measurements, the same calculation has to be repeated for each measurement, merely using other numerical values. If this work is performed by an electronic computer, the main advantage is that the results are available within about one hour of the readings being taken. Consequently better use can be made of the test equipment. Depending on the nature of the machine being investigated, the computer program can be made up of a number of sub-routines. The formation of the various sub-routines is described in the present article.

WHEN THERMAL turbo-machines are being investigated, measurements are taken of quantities, from which it is not generally possible to obtain a direct assessment of the machine. This involves a number of calculations, normally referred to as "evaluation".

Object of the Calculations

These calculations should provide the results for the following problems:

1. Many of the interesting quantities cannot be read direct, but must be determined from other measurements, i.e. by an indirect method. Thus, for instance, to measure the mass flow of liquids or gases the pressure difference at a throttling device, and the pressure and temperature before the device are measured. The mass flow can then be calculated from these readings and coefficients determined by calibration tests.
2. Other quantities, by virtue of their definition, can only be determined mathematically from other measurements. For example, the energy input to the gas in a compressor, or the energy extracted in a turbine have to be determined. This necessitates the observation of measurements at a number of points—at least two—on the machine, from which the change in the state of the gas can be followed. Then, by allowing for certain properties of the gas, the difference in energy can be calculated from the change in state.
3. From the individual quantities certain ratios or dimensionless quantities have to be derived. Thus, for instance, the efficiency is the ratio of the output power to the input power; or the load on the machine is measured by relating the energy converted in it to an energy referred to the peripheral speed of the rotor. The results of tests can be expressed more simply in this manner and are more generally valid, because they apply directly to "similar" machines. In this way the assessment of the investigated machine is simplified.

Advantages of Calculation by Machine

The calculations necessary for the evaluation of measurements, depending on whether the dimensionless quantities for the whole machine are required or whether they are also required for individual parts of the machine, can be very extensive. In conformity with the number of runs which have to be performed by the particular machine, these calculations have to be constantly repeated, using the same procedure but different numerical values. By employing a digital computer the following advantages are gained over calculating methods of a non-mechanized nature:

- The results of experiments are available much more quickly than when the measurements are evaluated "by hand". This is very important, because the test machine cannot be modified for a further series of measurements until the results of the first series have been evaluated.

- The accuracy of calculation is increased.
- The reliability of the calculation is improved.
- The cost of calculation is reduced, provided the cost of programming can be spread over a sufficiently large number of calculations.
- The staff liberated by changing over to mechanized computation can be employed for other, more interesting duties.

These considerations resulted in a change being made to the evaluation of experimental results by computer as long ago as the spring of 1957. To begin with the calculations were performed by an IBM 650 computer of the International Business Machines Centre, Zurich, later by the Siemens digital computer type 2002 installed in the Brown Boveri works in Baden.

Typical Application

A typical calculation of this kind is the evaluation of measurements on model machines. On these machines the effect of a large number of geometrical quantities on the characteristics of the machine, i.e. turbine or compressor, have to be determined. In view of the large number of experiments which have to be performed, the advantages of mechanizing calculation become particularly pronounced.

The principle is as follows: During the experiments, the readings are recorded on a report sheet. Immediately the experiments are complete, an input punched tape containing all the readings has to be made, which together with the tape giving the instructions for computation is read into the computer. On completion of the computation, a punched tape with the results is turned out, the actual table of results appearing in clear on the sheet printer. The time taken by the machine for calculation, including the input and output times, is about 10 minutes for 15 measuring points. A skilled engineer, calculating by hand, would require about two days for this work. Thus the interval between successive series of experiments can be reduced by two days. As a result of using the computer, the test equipment is better utilized, because far more experiments can now be performed in the same space of time. Moreover, according to a rough estimate, the cost of calculating by the computer is only about half the cost of calculating "by hand".

It is hoped that, at some later date, it will be possible to eliminate one of the intermediate steps described, when all measurements, instead of making the detour via the report sheet, are punched direct on the tape by a punch controlled by the measuring elements. Parallel with punching, the results would then have to be printed out, thereby providing a visual check.

Composition of the Computer Program from Sub-Routines

How must the computer program be organized in order that the above advantages may be enjoyed? Above all, it must be borne in mind that the experiments have to be performed on machines of very different kinds, i.e. turbines and compressors, which may be of axial or centrifugal design. For this reason alone, the arrangement of the measuring points and the procedure for evaluation is bound to be different. Furthermore, many of the experiments differ in their prime purpose: For instance, for guarantee tests only the main characteristic data are determined, such as the overall efficiency, and the throughput as a function of the load, while for tests in the development stage greater attention has to be paid to individual measurements.

One method is to prepare a separate program for each case, intended only for a definite machine and a definite purpose. In this case the cost of preparing the program and checking it are relatively heavy, especially when only a small number of measurements are involved. On the other hand the program can be organized in such a manner that it takes all possible conditions into account. But such programs become very large and unwieldy. For this reason it has proved preferable to collect identical parts of the program in sub-routines, which are already programmed and checked. They are stored in the library in the form of packs of punched cards, or as punched tapes, and can be combined to form a program adapted to the particular test object and experimental set-up.

Available Sub-Routines

The parts of the computer program suitable for combination as sub-routines will now be described.

1. Calculation of the mass flow of water or air from measurements obtained with the standard nozzle in accordance with the procedure laid down by VDI [1]. The formula given in these rules

$$\dot{M} = \alpha A_0 k \varepsilon \sqrt{2 \varrho_1 \, \Delta p}$$

applies to incompressible fluids and compressible gases. In it the following notation is used[1] (see also Fig. 1):

$\dot{M}$ = mass flow
α = coefficient of discharge* (including correction for roughness of the pipe-wall)
A_0 = area of the nozzle at ambient temperature
k = correction for temperature
ε = expansion factor
ϱ_1 = density of the fluid before the nozzle
Δp = pressure difference at the nozzle

The formula can be broken down into a factor K, which must be calculated alike for incompressible liquids and compressible gases

$$K = K\,(d, D, \lambda, t_1)$$

where d = diameter of the nozzle
D = diameter of the pipe
t_1 = temperature before the nozzle
λ = coefficient of superficial expansion (= twice the coefficient of linear expansion)

and a factor B, which expresses the nature of the fluid whose quantity is to be measured.

Incompressible $B_{in} = B_{in}\,(\Delta p, t_1)$

Compressible $B_c = B_c\,(\Delta p, t_1, p_1, \varkappa, X, D, d)$

in which $\varkappa$ is the ratio of the specific heats at constant pressure and at constant volume
X is the absolute humidity.

Then $$\dot{M} = K\,B$$

[1] All the notation used in the article is summarized at the end of the article, see page 329.

* This coefficient, as defined by VDI, is not identical with that defined according to American or British Standards, because it includes the velocity of approach factor.

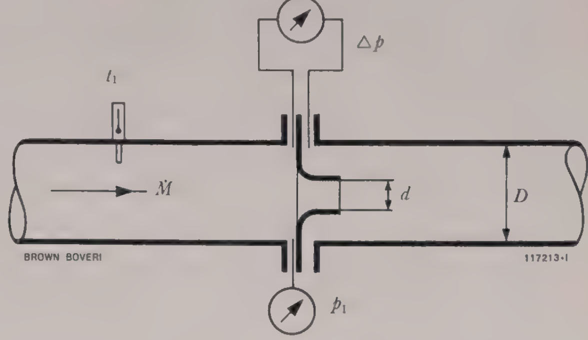

Fig. 1. – Arrangement of standard nozzle in the test pipe with the measuring points for determining the mass flow

For notation, see text.

Since the pressure before the nozzle is only used to calculate the quantity of compressible gases, it can be arbitrarily made equal to zero for incompressible liquids, and utilized for controlling the computer program (Fig. 2).

The functions determined experimentally

$$k = k\,(t_1, \lambda)$$
$$\varrho_W = \varrho_W\,(t_1)$$
$$\alpha = \alpha\left[\left(\frac{d}{D}\right)^2, D\right]$$
$$\varepsilon = \varepsilon\left[\left(\frac{p_2}{p_1}\right)^{\frac{1}{\varkappa}}, \left(\frac{d}{D}\right)^2\right]$$

where $p_2 = p_1 - \Delta p$, the pressure after the nozzle
and ϱ_W = the density of the water

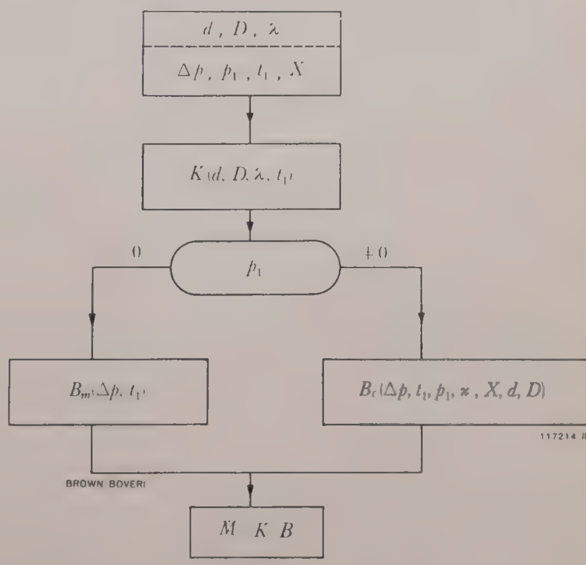

Fig. 2. – Flow diagram for computation of mass flow, sub-routine 1

For explanation, see text.

are represented in the program by approximation functions. This point will be dealt with in greater detail in one of the later chapters.

The flow diagram (Fig. 2) shows the computation procedure with the inputs and outputs. For the input data, those shown above the dotted line are characteristic of the measuring set-up and remain constant for a particular arrangement. Below the dotted line are the measurements which are different for every measuring point.

2. *To calculate the state* of air or combustion gases from a measured temperature and a measured pressure. It has already been mentioned that to calculate the energy put into or taken from a gas, the state of the gas must be known in at least two positions on the machine. By measuring two properties of state, e.g. the pressure and the temperature, the state of the gas at the particular point is unambiguously determined. But often the gas velocity at the point is so high that detecting elements mounted at a definite point record different values (owing to the flow in front of the measuring element being retarded and compressed) to those which would be recorded by detectors entrained by the flow. Thus the true values have to be obtained by correcting the measured values, using correction factors which indicate how much of the dynamic head is included in the measurement. The correction factor is unity when the gas is retarded to stagnation.

For the temperature sensing element this is known as the dynamic correction factor

$$\xi = \frac{T_m - T}{T^* - T}$$

where T = temperature of the gas
T_m = measured temperature
T^* = temperature at the stagnation state

and, accordingly, for pressure measurements, the dynamic correction factor of the pressure sensing element

$$\vartheta = \frac{p_m - p}{p^* - p}$$

with corresponding meanings for the various symbols. These correction factors have to be determined by calibration tests and may be regarded as constant within a certain speed range.

To correct the measurements it is also necessary to know the velocity at the measuring point and the enthalpy difference Δh corresponding to the kinetic energy, in order to be able to calculate the differences $T^* - T$ and $p^* - p$.[2] To accurately determine the velocity at the measuring point, at least one further quantity, e.g. the stagnation pressure, would have to be measured. Often it is quite impossible to install all the necessary detecting elements at one point or at comparable points, and in any case they complicate the layout of measuring equipment. Consequently, for the sake of the correction of the measured quantities, the velocity is often calculated approximately from the measured mass flow $\dot{M}$, the cross-sectional area A, and the state of the gas, provided the angle of flow α can be taken as known.

Hence this calculation requires those properties of state which have to be computed with the aid of the program. For the calculation by the computer it is most convenient to adopt an iteration method (Fig. 3). The measured quantities are regarded as a first approximation to the true property of state. With these an approximate value is calculated for the velocity at the measuring point and a first correction made to the measured values, which then act as the starting point for the next approximation. The iteration is discontinued when the improvement in the enthalpy difference Δh is less than a definite value ε from one step to the next.

Again in the flow diagram the input values given by the geometry of the measuring point and by the sensing element are shown separate from the measurements. With the latter the gas constant R must also be included, the value of which varies with the composition of the gas and the humidity.

3. *To calculate the integral of the change in enthalpy and entropy describing the change in state of air or combustion gases*

If the state of the gas is known in at least two places, the quantities can be calculated which indicate how the state changes. The most suitable are

[2] To calculate $T^* - T$ at given temperature T and given enthalpy difference Δh, the sub-routine 3.3 described later may, for instance, be used. The pressure ratio p^*/p can be calculated from the entropy difference at constant pressure corresponding to these temperature limits, using the sub-routine 3.2, also described later.

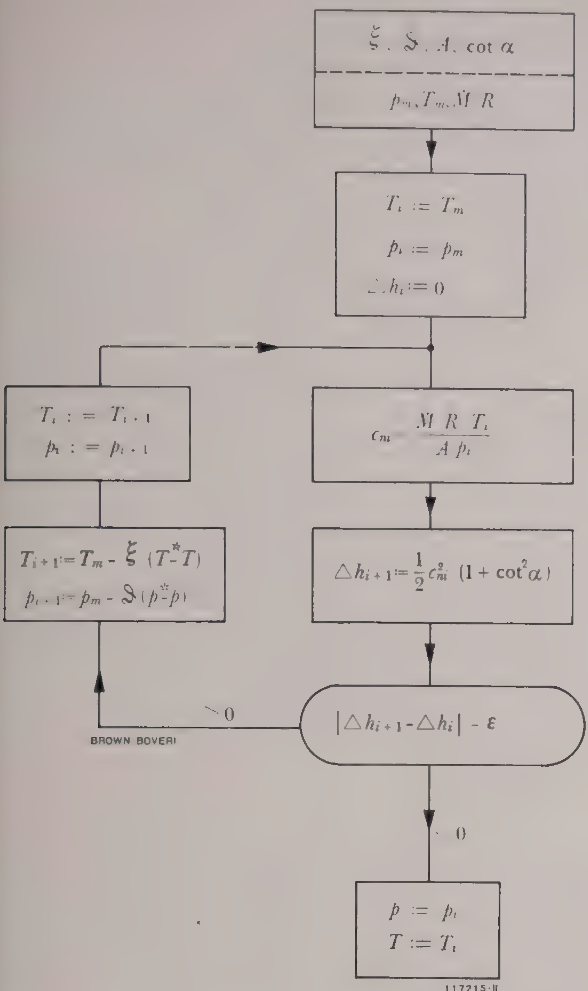

Fig. 3. – Flow diagram for computation of the gas state, sub-routine 2

For explanation, see text.

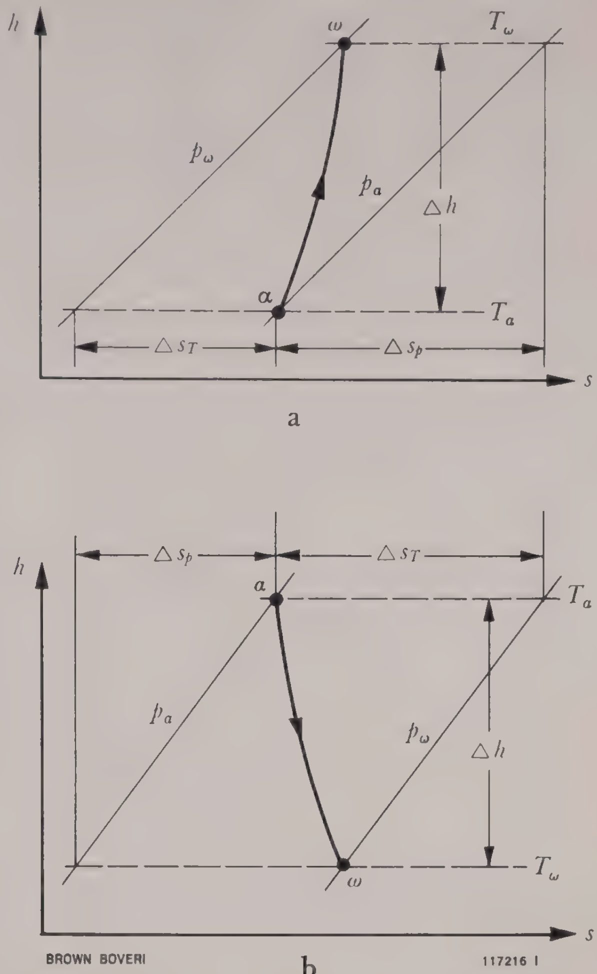

Fig. 4. – Differences in enthalpy and entropy in the enthalpy-entropy diagram

a: For compressors b: For turbines

For notation, see text.

the enthalpy difference Δh and the entropy difference at constant pressure Δs_p. If the gas may be regarded as an ideal gas, these differences can be expressed by the following integrals

$$\Delta h = R \int \frac{c_p}{R} \, dT$$

$$\frac{\Delta s_p}{R} = \int \frac{c_p}{R} \frac{dT}{T}$$

Therein c_p is the specific heat at constant pressure. The enthalpy difference indicates directly by what amount the energy of the gas was increased by work or the addition of heat, or by a change in the kinetic energy. With the aid of the entropy difference the polytropic efficiency

$$\eta = \frac{\dfrac{\Delta s_T}{R}}{\dfrac{\Delta s_p}{R}} = \frac{\log_e \dfrac{p_\omega}{p_a}}{\displaystyle\int_{T_a}^{T_\omega} \frac{c_p}{R} \frac{dT}{T}}$$

can, for instance, be calculated (Fig. 4) [2]. The subscripts α and ω denote that the quantities refer to the states α and ω. The above definition of the efficiency applies to compressors; for turbines the reciprocal must be inserted. From this definition it will be seen that it is convenient to calculate the quotient of entropy difference divided by the gas constant, instead of the entropy difference itself.

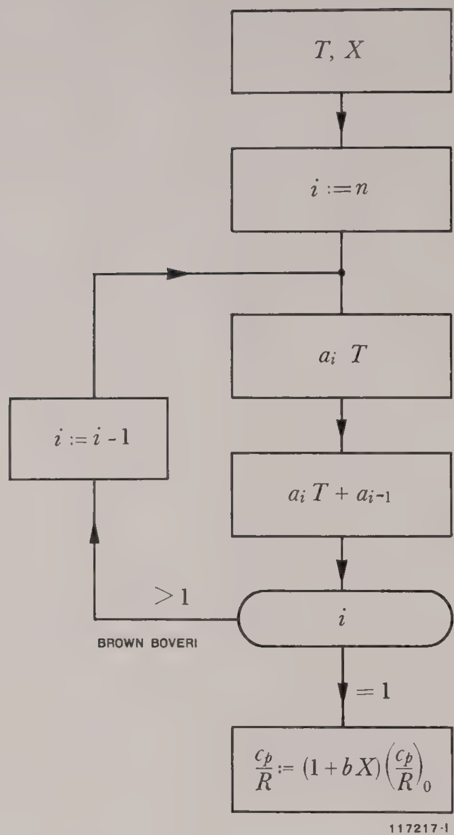

Fig. 5. – Flow diagram for computation of the specific heat by an approximation polynomial function, sub-routine 3.1

For explanation, see text.

In addition to these differences, however, it must be possible to work out one temperature limit when the value of the difference and the other temperature limit are known. In many cases an enthalpy difference can be determined more accurately from a measurement of mechanical power and the mass flow, for example, than from a direct measurement of the temperature difference.

The two integrals for the differences in enthalpy and entropy depend on the composition of the gas and the temperature since the specific heat for a particular gas is only a function of the temperature, at least within normal pressure limits. When calculating by hand, the values of these integrals can be taken from the curves applying to a certain gas composition or from tables. For the calculation by the computer the specific heat at constant pressure for a particular gas will be represented by an approximation polynomial having the form

$$\frac{c_p}{R} = a_0 + a_1 T + a_2 T^2 + \ldots a_n T^n = \sum_0^n a_i T^i$$

Closed integration of such a polynomial is possible, the values of the two integrals being calculated with appropriate constants, as required.

If the calculation is performed for air on the one hand and for the products of a stoichiometric combustion of definite hydro-carbons on the other, the integral values for any mixtures of these gases can be linearly interpolated, provided the mixing ratio is suitably defined. The calculated values then have to be corrected for the moisture contained in the air.

In the remarks that follow, only one gas will be considered, in order to simplify matters. The computer program must now be able to supply the value of the integrals with definite temperature limits, as well as one temperature limit for a definite value of the integral with a known value for the other temperature limit. To cope with these two tasks the program is best divided into the following three sub-routines.

3.1 For a given value of the temperature the specific heat divided by the gas constant is calculated for the dry gas at constant pressure $(c_p/R)_0$. This is best done by a cyclic process, in which for each run, an additional term is included in the polynomial having the following form (Fig. 5):

$$\left(\frac{c_p}{R}\right)_0 = [(a_n T + a_{n-1})\ T + a_{n-2} \ldots]\ T + a_0$$

This value finally has to be corrected for the humidity X.

3.2 Using a very similar method the values of the integrals are calculated for two given temperature limits. The choice of the computation procedure for the desired integral can, for instance, be governed by a quantity z, to which different values must be assigned in the two cases. For the calculation of the enthalpy integral it must be 0, while for the entropy integral it must be 1 (see Fig. 6). Here too the integrals have to be corrected for the humidity X.

3.3 If, for a given enthalpy or entropy difference and one given temperature limit, say T_a, the other limit T_ω has to be calculated, a decision must first be made ($z = 0$, $z = 1$) as to which of the two

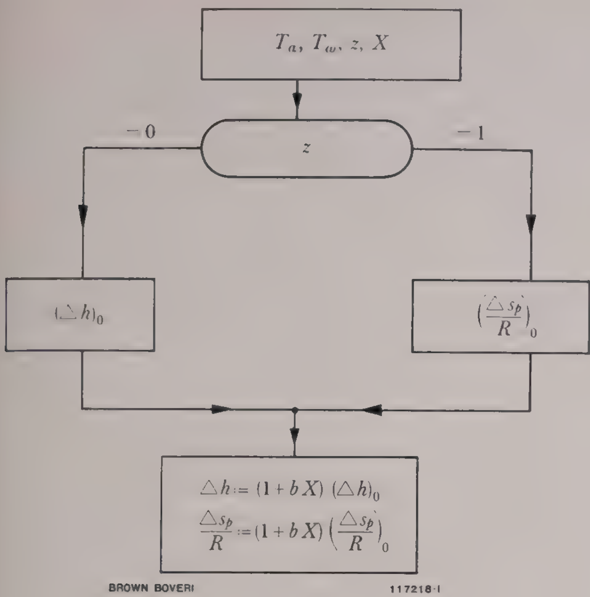

Fig. 6. – Flow diagram for computation of the integrals of the changes in enthalpy and entropy, sub-routine 3.2

For explanation, see text.

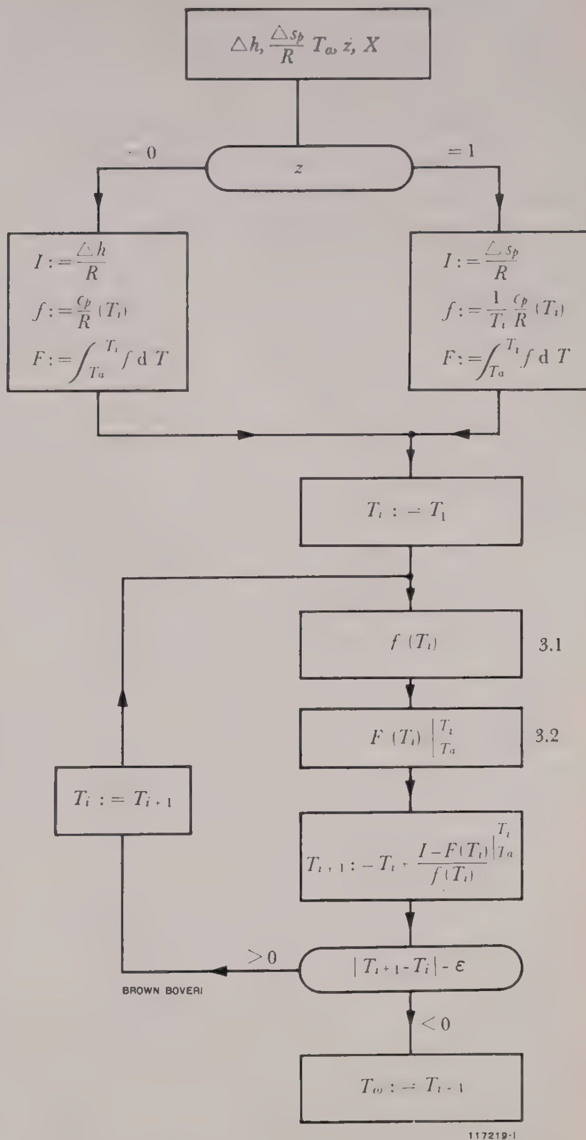

Fig. 7. – Computation of one temperature limit when the other limit is given, for given enthalpy or entropy difference, sub-routine 3.3

For explanation, see text.

differences is actually given (Fig. 7). The temperature limit T_ω is found when the given difference I equals the integral F between the given limit T_a and the desired limit T_ω. To find the zero point of the function $(I - F)$ dependent upon the desired temperature it is best to use an iteration. A better approximation T_{i+1} is obtained from the preceding T_i if the functional value $(I - F)$ for this temperature is divided by the derivative of this function for the temperature T_i:

$$T_{i+1} = T_i + \frac{I - F(T_i)\,\Big|_{T_a}^{T_i}}{f(T_i)}$$

This is commonly referred to as Newton's method and is ideal in this case because the derivative of the integral, i.e. the integrand f, can be calculated with sub-routine 3.1, and the integral F with 3.2. With appropriate specification of the initial value T_1 the method converges in the whole temperature range. If the improvement in the temperature from one step of the iteration to the next is smaller than a definite limit ε, the iteration is discontinued and the last calculated approximation for the temperature limit is the result.

Storage of Experimentally Determined Functions in the Sub-Routines

Within the calculations described above it is often necessary to determine quantities bearing an experimentally determined relationship to the calculated quantities, for instance in sub-routine 1 the coefficient of discharge $\alpha[(d/D)^2, D]$, the expansion factor $\varepsilon[(p_2/p_1)^{1/\varkappa}, (d/D)^2]$ and the density of the

water $\varrho_W(t_1)$; and in sub-routine 3.1 the function $c_p/R\ (T)$. It must therefore be possible to determine the value of these functions within a certain range for any value of the argument. This task frequently occurs in other calculations too; examples will therefore be given to demonstrate how it was solved in connection with the present sub-routines.

For non-mechanized calculation such functions are usually represented by tables or curves. On changing over to machine computation there are two fundamental possibilities for storing such functions within the computer program.

1. Associated pairs of values for the argument and the function are stored as a table. In most cases, though, the value of the argument for which the functional value is desired is somewhere between the stored values. Consequently a program must also be provided for interpolation of the functional values.
2. For each value of the argument the value of the function is calculated according to a program, provided the relationship can be expressed at least approximately as a formula.

Since the sub-routines are used in very different situations, the range in which the functional values have to be determined is very large. In order to fulfil the requirement for high accuracy of calculation and minimum storage capacity under these circumstances, the function was stored in every case in the form of a program operating on the principle of an approximation formula.

But in every case this approximation formula has to be found first. Often it is possible to abstract the physical process to be described in such a manner that only the significant factors are taken into consideration, and all insignificant factors disregarded. In many cases this physical model can be described mathematically. Such formulae expressing the physical process direct can then be utilized as the basis of the program.

An example is the calculation of the expansion factor for sub-routine 1. According to the VDI definition, ε is given by the formula

$$\varepsilon = \frac{\alpha_k}{\alpha}\sqrt{\frac{1-\mu_k^2 m^2}{1-\mu_k^2 m^2 P^2}}\sqrt{\frac{P^2}{1-P^\varkappa}\,\frac{\varkappa}{\varkappa-1}\,(1-P^{\varkappa-1})}$$

in which

μ_k = contraction factor for gases

α_k = discharge coefficient for compressible gases

α = discharge coefficient for incompressible liquids

$\varkappa$ = ratio of the specific heat at constant pressure to that at constant volume

$m = \left(\frac{d}{D}\right)^2$ = ratio of the cross-sections of nozzle and pipe

P = abbreviation for $\left(\frac{p_2}{p_1}\right)^{\frac{1}{\varkappa}}$

A closer examination shows that the value of the expansion factor is mainly dependent on the quantity P and on the cross-sectional ratio m, but only to a very small extent on the ratio of the specific heats which otherwise appear at various points in the formula. Thus, for example, with $\varkappa$ between 1·0 and 1·5, the differences in ε remain below 0·13 %. If we were now to expand the third factor in powers of $(1-P)$, the ratio of the specific heats can be arbitrarily made $= 1$ in the various terms of the expansion. The series may be discontinued after the third term without the accuracy suffering. Furthermore, for the standard nozzle, ignoring radial expansion, the contraction factor μ_k and the ratio of the discharge coefficients α_k/α may both be equated to 1. Thus the simplified formula

$$\varepsilon = \sqrt{\frac{1-m^2}{1-m^2P^2}} \cdot \sqrt{1-\frac{3}{2}(1-P)+\frac{1}{3}(1-P)^2+\frac{1}{12}(1-P)^3}$$

is obtained, which is ideal for cyclic programming. The deviation of the values calculated with this formula from those given by curves plotted from measurements and reproduced in the VDI regulations is less than 0·14 %, provided the cross-sectional ratio is restricted to $m \leqslant 0{\cdot}6$.

There are, however, other cases in which it is either impossible to find a model for the physical process, or the formulae derived from a model are too complex and cumbersome to permit their use in a computer program. In such cases an attempt must be made to find a suitable approximation formula which approaches sufficiently closely to the given points but bears no direct relationship to the physical

process. But there is no point in forcing the function to pass through the given points, which are often affected by errors of measurement in any case. The approximation function would only be rendered more complicated by doing so. Using the method of the smallest squares it is usually relatively easy to find a simple approximation formula.

This method was used, for instance, to calculate the values of the specific heat at constant pressure divided by the gas constant c_p/R as a function of the temperature for sub-routines 3.1. The tabulated values (see [3, 4, 5] for example) were mostly obtained using statistical formulae of quantum mechanics from analysis of the spectra of the various gases (e.g. [6 and 7]). However, these formulae are far too complex to be used as a basis for a sub-routine in the computation. In contrast, it was possible to find a second-order polynomial as an approximation to this function for dry air in the temperature range $240\,°K < T < 580\,°K$, and a fourth-order polynomial for dry air and the products of a stoichiometric combustion in the range $580\,°K < T < 1280\,°K$, the deviations from the tabulated values being less than 0·03% over the entire temperature range. Representing the c_p/R function by approximation formulae has, as already mentioned, the additional advantage of permitting closed integration of the polynomials, so that the integrals can also be calculated by approximation polynomials with appropriate coefficients.

Conclusions

The present article has shown how great and varied are the advantages gained by the introduction of digital computation for the evaluation of measurements. But it is important not to program merely the procedure normally used when calculating by hand; rather, the problem must be tackled in a way which suits the abilities of the computer. This applies particularly to the calculation procedure for which, corresponding to the very high speed of calculation, cyclic and iterative methods are often employed, whereas explicit methods would be used for calculation by hand. Likewise for storing experimentally determined functions and for the build-up of the whole program it is essential to find a suitable procedure. To explain this point a description of existing sub-routines was given, which allow programs to be compiled in a very short time. It thus becomes possible to employ a computer rationally even for the evaluation of quite small series of measurements on different kinds of test objects.

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Summary of the Notation Used in the Text

Symbol	Meaning	Units
A	Cross-sectional area	m²
B	Factor in the formula for mass flow	$kgs^{-1}\,m^{-2}$
D	Diameter of pipe for measurement of mass flow	m
F	Integral value $\int_{T_a}^{T_i} f\,dT$	°K or dimensionless
K	Factor in the formula for mass flow	m^2
$\dot{M}$	Mass flow	kg/s
P	Abbreviation for $(p_2/p_1)^{1/\varkappa}$	—
R	Gas constant	J/kg °K
T	Absolute temperature	°K
X	Molecular ratio of the water vapour in the air to dry air	—
a	Polynomial coefficient	—
c_p	Specific heat at constant pressure	J/kg °K
d	Diameter of the standard nozzle	m
f	Function c_p/R or $\frac{1}{T}\,c_p/R$	dimensionless or 1/°K
h	Specific enthalpy	J/kg
k	Temperature correction for nozzle area	—
m	Ratio of areas of nozzle and pipe	—
p	Absolute pressure	N/m^2
s	Specific entropy	J/kg °K
t	Temperature above the freezing point of water	°C

Symbol	Meaning	Units
α	= Discharge coefficient for standard nozzle in a pipe roughened by normal service	—
ε	= Expansion factor	—
η	= Polytropic efficiency	—
ϑ	= Pressure detecting element dynamic correction factor	—
$\varkappa$	= Ratio of specific heats	—
λ	= Coefficient of superficial expansion	1/ °C
μ	= Contraction factor	—
ξ	= Temperature sensing element dynamic correction factor	—
ϱ	= Density (specific weight)	kg/m³

Suffixes and symbols

0 = at ambient temperature
$_0$ = for dry gas
α, ω = denoting different states
D = of the nozzle
1 = before the nozzle
2 = after the nozzle
i = current suffix
in = incompressible
c = compressible
p = at constant pressure
T = at constant temperature
W = of water
Δ = difference

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THE EMPLOYMENT OF DIGITAL COMPUTERS IN THE DESIGN OF ELECTRICAL MACHINES

681.14:621.313.001.24

A useful aid for the calculations involved in the design of electrical machines is the digital computer, which can be entrusted with frequently repeated routines, when fed with appropriate programs. Brown Boveri have been preparing such programs since 1957, many of them being repeatedly utilized in practice.

PROGRAMMED calculation of electrical machines is one of the special applications of the digital computer. The calculations involved are required when designing electrical machines, as well as for checking constructional details. Drafts and the calculations for such units as transformers, motors or generators are continuously recurring in practice and take up a great deal of time. A design calculation necessitates the employment of a large number of formulae and laws, possessing general validity in the range of a certain type of design, but which to be numerically evaluated individually every time. Owing to its ability to perform extensive numerical calculations, following a preset program, at high speed, the digital computer is an ideal aid for such design calculations.

Design Calculation and its Automation

As a general rule the design calculation of an electrical machine may be divided into three phases: dimensioning, determining the properties, and assessment. When the calculation is carried out by hand, these three activities are interlaced to a large extent. The skilled design engineer is continually assessing the intermediate values and results, with the object of recognizing the effect of the dimensions adopted as quickly as possible. Also the procedure for a manual calculation will differ from one case to another. It is obviously a process which does not follow any hard and fast plan. The main object of automatizing such processes is therefore planning an organized general schedule for the calculation. The procedure must be moulded into a definite succession of rational steps. Furthermore, the task in question, if it is to be handed over to a program-controlled computer for solution, must be suitable for formulation. Therefore it is necessary to be able to analytically express graphical processes and inevitable estimates based on experience.

All these requirements are essential stipulations which have to be fulfilled when a design calculation by hand is converted into an electronic computation. In many cases the task is far from easy, but nevertheless soluble. Having planned the structure of the computation process and formulated the quantities to be calculated, the use of the electronic computer opens up possibilities offering numerous advantages over calculation by hand.

The Calculation Program

The program for a design calculation comprises all necessary constants and data, as well as the relevant instructions, the execution of which in the specified order represents the computation procedure for a particular type of machine. The mathematical operations for which instructions are given process the constants of the problem, as contained in the program, and the variable data which are features of the design in question. The instructions are so arranged

that they can be carried out in the manner appropriate to every possible design. The instructions and all data not applying exclusively to the particular application consequently form a generally valid whole, known as the program. Thus each program, since it possesses general validity in its field of application, only needs to be set up once. The instructions and general numerical values are recorded in code form on punched tape. The program can be retained in this permanent form and is always available for communication to the stores of the computer. Before such a program can be put into action, the variables have only to be compiled.

Depending on the content and the extent, computer programs may be divided into three categories, as follows:

1. Those which determine the properties of a design whose geometric proportions are completely given.
2. Partly automatic programs which supplement designs existing in an incomplete state and determine their properties. Partial dimensioning is achieved by observing conditions relating to the utilization and the behaviour of the finished design, and in some cases to prevailing standards, and by fulfilling these conditions in the best possible manner.
3. Completely automatic programs, which carry out the whole draft in such a manner that the resultant solution fulfils all stipulations made regarding the properties.

A feature common to all these programs is the determination of the properties of the design. The difference between the three categories is in the existence and the extent of automatic dimensioning. The actual process of drawing up the design makes allowance for, and fulfils as well as it can, the conditions which can be stipulated for the design to be supplemented or drawn up from scratch. This process of aimed design is also known as optimizing, since it aims at providing the best solution, aided by programmed assessment of certain results in conjunction with appropriate modification of the parameters.

In accordance with the graduated capabilities, the types of programs listed naturally also differ in the time taken to set them up, in their structure and extent, and finally in their behaviour. With the degree of automation, the elegance with which each program can be executed increases. Whereas a completely automatic program determines the best solution in a single sequence, the simple 'check' program and partly automatic programs have to work out several variants before a good solution is found. However, the higher the degree of automation, the more work setting up involves, the greater the need for storage capacity and the greater the volume of actual calculation. The flexibility of a computer program is influenced in the opposite sense. With programs having a lower degree of automation, the flexibility is greater, because with them the suitability of the solution is left to human judgement. Without changing the program it is possible for the manner in which the assessment is made to change. The chance of changing a program also diminishes with increasing automation. Completely automatic programs are consequently only employed for revolutionary types of machines.

Preparation and Employment of a Design Program

In Fig. 1 the steps leading up the completion of a program, and during its operational employment are depicted schematically. The upper leg in this diagram refers to the preparation of the program. In a single operation the calculation formulae and all necessary mathematical and technical constants are translated into the language of the computer. The now encoded program is next punched on the tape with the aid of a sheet printer which also acts as a punch. Thus, at the same time as the punched tape is being produced, a record in clear is typed out showing the contents of the tape. Only when the tape is finished is it possible to impart the instructions and general data to the stores of the computer. Since the program possesses general validity within its field of application, it is

1 = Computer
2 = Input
3 = Output (punch)
4 = Sheet printer
5 = Punched tape
6 = Record in clear
7 = Program library
8 = Program
9 Calculation formulae
10 = Universal and physical constants, general technical data
11 = Design department
12 = Variable machine data
13 = Result

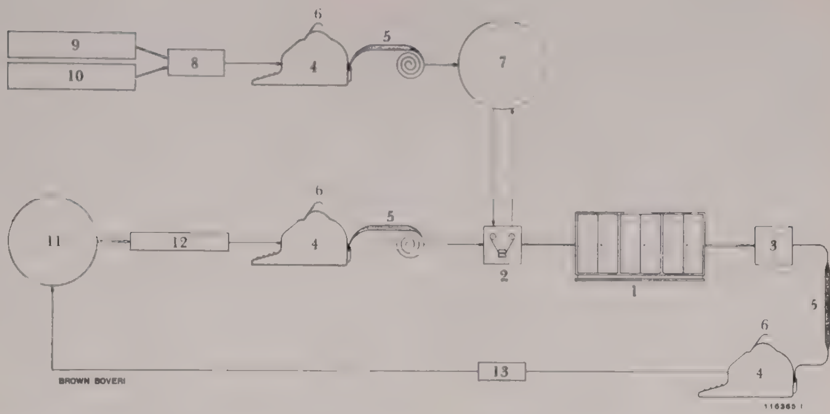


Fig. 1. – Diagram showing the preparation and employment of a design program for the digital computer

The upper leg shows the preparation of the general computer program, which is stored in the form of punched tapes in the library of the Computer Centre. The lower part of the diagram shows how the program is fed into the computer and the problem calculated. The design engineer collects the various performance data of the machine to be calculated, which are read into the computer together with the existing program, all in the form of punched tape. On completion of the computation, the results are passed back to the design department.

kept in the library of the Computer Centre in the form of a punched tape and is thus available for use again at any time.

The lower leg of the diagram in Fig. 1 applies to the preparation and execution of the calculation. The responsible engineer from the design department compiles the variable data for one or more design variants. These data contain the ratings of the electrical machine being calculated and, depending on the nature of the program, further requirements relating to the form of the design or the attainment of certain properties. The variable data are transferred to a punched tape by a sheet printer, and that completes the preparations for the calculation.

Only the succeeding operations are performed in the Computer Centre. They can be performed by the operator without any assistance by the responsible engineer. The program kept in the library and the variable data are fed into the computer store through input elements. Then the computer performs the instructed operations with the given numerical values, the result appearing as a punched tape. Independent of the computer the results can be translated into clear by the sheet printer. In this form they can be handed over to the design department.

The Uses and Limitations of Electronic Computation

Program-controlled design calculation offers a variety of advantages over the classical method of calculating by hand. When setting up a computer program the methods of manual calculation are not exclusively translated into the computer language, as a general rule. In view of the high speed and the large storage capacity, the amount of actual calculation work can be greatly increased. The programmed task may therefore be based on more extensive and refined methods of solution, which benefits the calculation, in that it is thereby made more comprehensive and more accurate. In many cases the use of the digital computer has made possible the employment of methods involving far too much work to allow them to be used in manual calculations.

By means of the program a great deal of mathematical work is transferred to the computer, which previously was the entire responsibility of the design engineer. This work is performed rapidly and free from errors. By way of example, the normal calculation of a turbo-alternator used to occupy two full working days, when done by hand. With one of our semi-automatic programs this work can be completed

within five minutes, disregarding the time taken to feed in the program. The engineer, now largely relieved of the burden of exhaustive calculations, is thus able to devote more time to a thorough criticism of the results. To judge by experience gained so far, he utilizes the greater part of time saved in thinking out refinements to the design, so that changing over to electronic calculation does not represent any great reduction in the number of design engineers, but an improvement in the quality of the calculated product. In fact, the high speed of the digital computer allows several variants to be calculated in a very short time. It provides a means of selecting the best design from several in every case; this was not always possible with manual calculation owing to the time and the amount of work involved. With transformers and rotating electrical machines it has been established that electronic computation has resulted in a appreciable improvement in the quality. This may be expressed either by an improvement in the operating properties or a reduction in the weight, depending on which is considered more valuable.

Generally speaking, it may be claimed that the change from manual to electronic calculation has opened up numerous new possibilities. Moreover, design programs are very versatile in their application, and suitable for work in connection with tenders, isolated new designs and the compilation of price lists.

A technical limit to the scope of electronic design calculations is the storage capacity of the particular computer. Now one of the features of design programs is the large number of instructions and logical operations, and the variety of the data required and individual results. The storage capacity needed for our programs, as set up for the Siemens 2002 computer for instance, is between 3000 and 11 000 cells, the largest capacity being required for the completely automatic programs. Thus, to undertake design calculations without any limitations on the scope of the task it is essential to employ digital computers having a large storage capacity.

Economics of Electronic Computation

In this connection it is equally important to consider the economics of electronic design calculation. The work involved in setting up a program-controlled calculation must be compared with the attainable usefulness. It must be pointed out that electronic computation, with its associated advantages, is only made possible by the fairly expensive business of preparing the program. Depending on the degree of automation, it may take from 500 to 3000 man-hours – a not inconsiderable figure – to prepare a single program, quite a high proportion of which can be left to suitably trained auxiliary personnel though. The fixed costs must be offset against the attainable improvement in the quality of the product and reduction of its price, in addition to the saving effected in the engineer's time, if electronic computation is to justify itself as an economical process. But the more often a program is utilized, the smaller each variant's share of the initial costs becomes. An economic gain is therefore assured when the extent of a program is adapted to the frequency with which it will be used and the possible improvement in the quality of the calculated product. In contrast, the use of an electronic computer is uneconomical when the cost of preparing a program exceeds the cost of calculation with simpler means.

Development and Present State of Electronic Design Computation at Brown Boveri

Commencing in about 1957, exhaustive studies were carried out to investigate the possibilities of using program-controlled computers for calculations needed for the design of electrical machines, a number of different computers being tried out to establish their suitability. In the course of these investigations and subsequently our first design programs were prepared for the different types of computer which were available. Even at that time some of these programs were usable for practical calculations. The experience gained then proved most useful when it came to the preparation of programs for our own computer. An early start was made with the conversion of existing programs to suit the computer decided upon, and to prepare new programs for it. Thus by the time the digital computer was ready to commence service several design programs were already available for practical utilization. Since then,

of course, others have been added. Now the library contains tried programs in different degrees of automation for numerous electrical machine designs. They are constantly being used for practical purposes and have rendered valuable service in the preparation of tenders, isolated new designs and the compilation of price lists.

Induction machines

In the field of induction machines partly automatic and check programs are being employed with great success. Programs for slip-ring machines, squirrel-cage motors with double-cage rotor, or with deep-slot rotors, have existed for some time. In these programs, apart from determining the electrical, magnetic and thermal conditions, stress is laid on the determination of the operational performance. A program serving this purpose will be described briefly in the next chapter.

Synchronous machines

For the field of synchronous machines there are efficient, partly automatic programs for salient-pole generators of low output and for large hydrogen-cooled turbo-alternators. Apart from their use for the automatic dimensioning, these programs are employed to calculate all the major properties. For the high-power machines in particular, a detailed price and weight calculation is of great economic value. It has therefore been included in the program for turbo-alternators and permits a comprehensive assessment to be made of the solution.

Transformers

For small transformers with ratings of up to 650 kVA a program has been drawn up with the main object of gaining experience. It can be used for price list calculations and for variants on a basic design. It was used in the development of a new range of units. Afterwards a new program was devised to cover the whole range from 20 to 6000 kVA, provided no regulating winding is involved. It is at present being introduced, both for the development of new series and for dealing with quotations and orders.

Apart from these typical design programs, there are also several programs for special purposes, such as the calculation of short-circuit forces,[1] magnetic fields, the current distribution in parallel groups of coils, and the distribution of impulse voltages.

At the present moment the program-controlled calculation of electrical machines is being rapidly expanded at Brown Boveri. Numerous other programs with different degrees of automation are being prepared. From the success gained so far it is evident that electronic computation will in due course find its way into all branches of engineering design within the Company.

Description of a Partly Automatic Program for Calculating Squirrel-Cage Motors

Task of the Program

With the program it should be possible to determine the electrical, magnetic, thermal and starting properties of any squirrel-cage motor of given constructional design. In the event of the design department not prescribing a definite stator winding, the program should also be able to find the best stator winding and carry out the calculation for it.

Principle of the Program

When setting up a program, a block diagram is first prepared, showing the various stages of the calculation in very general terms. Partial problems which are complete in themselves are denoted by a box containing a keyword (see Fig. 2). The next step consists of working out these partial problems in detail, thereby producing a flow diagram. This is then a complete set of instructions which finally has only to be translated into the language of the electronic computer. Let us now examine the block diagram, with reference to Fig. 2: On starting the computation the motor data written on punched tape are first transferred to the store of the computer. They contain the stipulated operating conditions and the constructional data of the motor. The first

[1] M. Christoffel: Calculation of short-circuit stresses in transformer windings with the aid of digital computers. Brown Boveri Rev. 1960, Vol. 47, No. 5/6, p. 321–8.

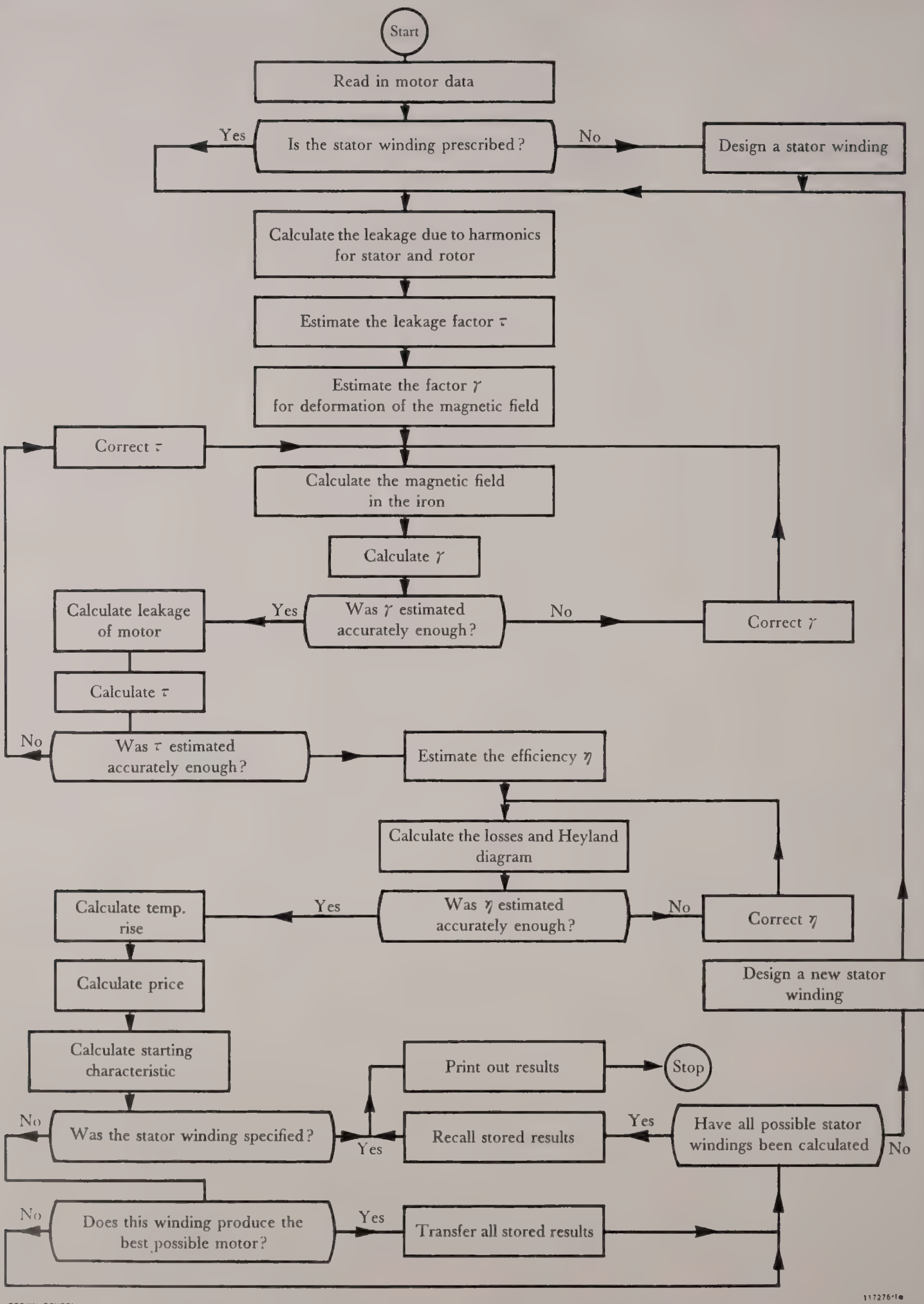


Fig. 2. – Block diagram for the computation of squirrel-cage motors

Rectangular boxes denote closed sub-routines. Those with rounded ends denote decisions to be made by the computer. Note the links at three points. These indicate that parts of the calculation have to be repeated several times because a certain factor is unknown to begin with and must consequently be estimated. Finally, when the factor can be calculated, the accuracy of the estimate can be judged. If the accuracy is inadequate, an improved value is estimated with the aid of the calculated factor and the calculation repeated.

step is to decide whether these data determine the stator winding. If they do, the design of the motor is already fixed and the task of the program is solely to determine its properties. If not, the computer has to decide on the data for the missing winding, according to a separate instruction, and only then can it carry on with the determination of the properties. First a Fourier series has to be evaluated to obtain the leakage due to harmonics of the stator, followed by another infinite series to determine the same leakage for the rotor.

To enable the magnetic field in the motor to be carefully investigated, and from it the no-load and short-circuit currents, two coefficients γ and τ are required, whose magnitude depends on the saturation of the iron and the constructional design. The stray factor τ summarily allows for the stray field in the motor; the factor γ describes the deviation of the field from the sinusoidal form when the iron is heavily saturated. Neither of them is known to begin with and they are both obtained in the course of the calculation. They must therefore be estimated to start with. Having calculated the fild in the iron, γ can be worked out and compared with the estimated value. If the deviation is very great, a better value is assumed for γ and the calculation repeated. This procedure is repeated until the estimate and the calculated value agree sufficiently closely. Not until then can the stray field, and from it the factor τ, be calculated. The same procedure is adopted as for γ, and for the efficiency η. Finally the starting characteristics, i.e. the curves of stator current and torque against the speed, are computed. If the stator winding was given, the calculation is now ended and the results are simply printed out in a list.

However, if the designer is free to use a stator winding of his own preference, he can leave the choice of this to the computer by not prescribing any relevant data in the program. The program then ensures that all possible forms of winding are considered successively, and the properties of each motor calculated. From the finite number of such solutions, the computer then has to select the best by comparing the price and the performance figures, such as temperature rise, efficiency, power factor, starting and stalling torque. Every time a better solution than all preceding ones has been found, the results are transferred to a place specially provided in the store. Here is always kept the best solution determined and, after calculating all possibilities, the data for the best motor.

If guarantee values are prescribed, these and the motor data are fed into the computer, which first checks whether they can be fulfilled. If any of the guarantee stipulations is not met, the results of the calculation are not stored and are erased on commencement of the next calculation. It is, of course, possible for the conditions stipulated for guarantees to be so severe that there is no winding which can possibly fulfil them. In this case the computer prints out a short notice instead of results, indicating to the engineer all the information he needs to pass judgement on his design.

When calculating motors by hand, graphical methods (such as the Heyland diagram) are used which, with the aid of analytical geometry, can be made readily accessible for the electronic computer. The design engineer also employs numerous curves and tables which can also be provided in an analytical form by various means.

Practical Work with the Program

Having set up the program and checked it, the engineer deposits it in the Computer Centre with his instructions for its use. Each design engineer, in addition to receiving the set of instructions, also has a flow diagram showing the method adopted for computation, and a set of printed lists for input data, which he has to fill out each time before utilizing the program and hand over to the Computer Centre.

On receipt of an order or enquiry the design engineer first prepares a draft that can be easily calculated from existing types, from motors which have already been built, and utilizing his own experience. The data of his draft are entered in on a printed form, handed over to the Computer Centre, where they are transferred to punched tape and subsequently computed. All the work in the Centre is carried out by auxiliary personnel. It is of course possible for the engineer to remain in the Centre and observe the results as they come out of the computer, permitting him to repeat the calculation with different data. Without being present in the Centre he can also

leave instructions for several sets of data to be calculated and can compare the results afterwards. In this manner the design engineer can draft motors quickly whose details are matched in every respect, a process which would be quite impossible with the slide-rule, above all for very small machines. He is thus relieved of almost all manual work, and can concentrate his efforts on the utilization of his experience. This combined operation by the design engineer and the computer has proved very successful.

The time taken to feed in the design program described is in the region of six minutes. The calculation of one kind of winding lasts about three seconds. If the computer has to investigate one hundred different windings for a particular motor it requires roughly five minutes for this. Printing out the complete list of results, involving some 200–300 numbers and containing the prescribed motor data, lasts barely two minutes. A skilled engineer requires approximately one day for such a calculation. The program described is therefore an ideal means of working out several different designs of an induction motor and, by comparing the results obtained from each calculation, to find the best solution.

G. NEIDHÖFER
(KME) E. BAHM

IMPROVING THE SMOOTH-RUNNING QUALITIES OF TURBO-ALTERNATORS

681.14:621.313.322-81

The author describes the employment of the digital computer during the design of turbo-alternator windings consisting of bars made of hollow-drawn conductors. Irregularities arising during the manufacture of the conductors are balanced by using a computed bar assembly, thus eliminating one of the main thermal factors affecting the smooth-running qualities of such rotors.

Temperature Rise and Smooth-Running Qualities

QUITE INSIGNIFICANT differences in the temperature of diametrically opposed zones in the iron of turbo-alternator rotors can produce distortion seriously affecting the smooth running of the machine. Such deviations from the normal temperature rise in operation are caused by thermal disturbances originating in the excitation winding situated in the rotor slots. The generation of unequal amounts of heat in the slot filling due to the passage of current, and the different rates of dissipation of the heat from the copper are the main disturbances. When electrically loaded, the local temperature rise of the rotor iron is always influenced by the adjacent slot. If the passage of current through diametrically opposed groups of slots produces different temperature rises in the surrounding iron, this results in distortion of the rotor body which, when running at speed, is apparent as a deterioration in the smooth running of the alternator.

The Hollow Conductor as a Source of Disturbance

In the Company's turbo-alternators for medium and high outputs most of the active part of the rotor winding consists of hollow copper conductors, the coolant flowing through the cavity and the metal acting as the conductor of electricity. With this direct system of cooling the rotor iron is in thermal contact with the insulating lining, and thus indirectly with the bars in which the heat is generated when the machine is running. Any differences between the hollow conductors allocated to the various slots can therefore lead to unequal peripheral temperatures in the iron when current is flowing through the conductor.

Owing to the irregularities inherent in the manufacturing process, there are indeed differences between individual hollow-drawn copper sections, which may adversely affect the temperature rise in service. On account of the essential manufacturing tolerance, the cross-section of bars composed of such conductors (hereafter referred to as hollow bars) may consequently not be exactly alike, despite theoretical equality of their dimensions. Whereas the external dimensions of the bars vary in very close limits, the ratio of copper cross-section to cavity is not always constant. Differences between the actual contour of the cavity from one bar to another are quite small, and the roughness of the surface of the duct is practically the same in all bars. Along the length of a single conductor the cross-section varies imperceptibly, so that the differences originating during manufacture can only be expressed by the ratio of the duct area to the copper cross-section.

An improvement in the dimensional accuracy of the drawing process, which is already difficult enough, could only be achieved by increasing the cost out of all proportion. The measures described later indicate that such a step is not even necessary, as it has proved possible to counteract the irregularities mentioned above.

According to the foregoing remarks and owing to the strict dimensional accuracy of the exterior of the hollow conductors, it is sufficient to regard either the

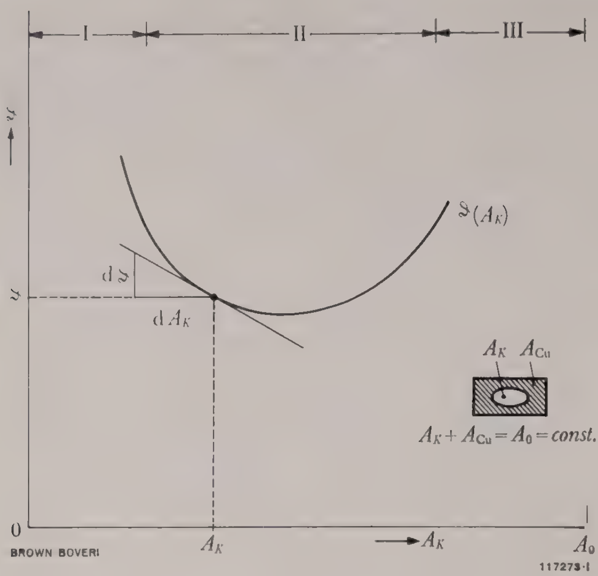


Fig. 1. – Temperature rise of a hollow conductor carrying current and cooled by a stream of gas, shown in terms of its cavity area A_K and the copper cross-section A_{Cu}

Zone I: low copper losses, heat dissipation too small

Zone II: moderate to high copper losses, adequate heat dissipation

Zone III: very high copper losses, strong but still inadequate heat dissipation

area of the gas duct or the copper cross-section as the sole variable property of the bar governed by its manufacture. Since the internal cross-sectional area of a tube can be determined directly with ease, and with adequate accuracy, every hollow conductor is hereafter characterized by the cross-sectional area of its cavity.

Cavity Cross-Section as Characteristic of the Bar

Differences in the size of the copper cross-section and the area of the gas duct affect the heat generated by the passage of current and the rate of cooling by the gas circuit. Two components—the rate at which the heat arrives at the surface and the rate of flow of the coolant through the duct—are together responsible for the temperature rise in a hollow conductor, but for such studies it may, under certain circumstances, be sufficient to know only one of the two cross-sections. There is in fact an interdependent relationship between the temperature rise in a hollow conductor and the size of its cavity, when the external dimensions remain unchanged and all other quantities in the system are constant. This relationship is clearly illustrated in Fig. 1. It is important to establish that, depending on the size of the cavity area, any change in it can cause either a positive or negative change in the temperature rise. Having regard to the excitation losses which are involved, the conductors of Brown Boveri alternators are designed to be within the left-hand section of the curve and not too close to the temperature minimum, so that for all applications the temperature change is in the same sense for all changes in the cavity area. On this assumption, and assuming there is not an undue amount of scatter in the cavity area from one tube to another, it is permissible to assume a linear relationship between changes in the cavity area and changes in the temperature rise, the relevant section of the curve being replaced by its tangent (see Fig. 1). Hence, under the above assumptions, it is admissible to dispense with the calculation of the temperature rise of every one of the large number of different hollow conductors, and to evaluate the bars alone on the basis of the deviation of their cavity area from an existing mean value. Furthermore, it is possible and sufficiently accurate to measure the quality of a slot filling according to its mean cavity area deviation because, according to the design on the drawing, every bar in the slot is supposed to possess almost the same thermal weighting.

Preventive Measures

It is possible to distribute the hollow conductors of different cross-section in such a manner between the individual slots that their different thermal effects cancel each other out, as nearly as possible. A prerequisite condition for this is the measurement of the characteristic value for every bar to be incorporated. By means of a systematic process of combination, groups of bars are formed which, according to the measurements obtained, should exhibit nearly the same behaviour when heated. The thermal effect of the residual deviations between the groups so formed can be almost eliminated by placing groups of bars having almost the same properties in diametrically opposed slots.

The method as such represents a precautionary measure which almost completely eliminates the

effect of disturbances resulting from manufacture, thereby rendering a major contribution to the improvement of the smooth-running properties of rotors equipped with hollow bars. By suitably organizing the process, its execution in practice involves very little extra time or work.

Description of the Calculation Process

The mathematical processing of the measurements obtained for the bars and carrying out the process of combination referred to above, together form a task, for the solution of which the digital computer is ideal. At Brown Boveri a routine has been used for some time, which was specially devised for the allocation of bars to the rotors of turbo-alternators in the course of manufacture. The computer is fed with the individual measurements and, apart from providing the quality factor for each conductor, yields a complete insertion programme which can be used by the workshop.

Principle of the Computer Program

Fig. 2 shows a block diagram illustrating the overall structure and the principle of the computer program. The remarks which follow are intended to give an explanation of certain parts of the problem—marked with numbers in boxes in the diagram. To demonstrate the process some of the more important steps are illustrated in Fig. 3. The term "item" used in the block diagram refers to all hollow conductors of the same cross-section and the same length. The slot filling of the turbo-alternators in fact comprises various conductors of different sizes.

Computation of bars (1) [1]

In box 1 the stored results of the measurements of the hollow conductors are evaluated and all necessary preparations made for the subsequent process of allocation of the bars.

The calculation of the characteristic value for each bar—in this case the cavity area—depends on the method used for measurement. Having regard to later operations, the bars are arranged according to the size of their cavity areas, and then stored. If the number of bars delivered exceeds the number required for assembly, first those bars are sorted out whose absolute cavity area deviation from the existing mean is extremely large. Next the mean area of the bars to be used is calculated. The above calculations are performed for each item (see also Fig. 3a).

First formation of groups of bars (2)

From the succession of bars, as they are now arranged, groups are formed which are later allocated to fill the respective slots (see also Fig. 3b).

In order to attain a good combination of bars without spending too long on the calculation, it is essential not to leave the first allocation of the bars to chance, but to carry it out according to a definite plan. The quality of each group is determined by the relative deviation of its cavity area from the mean value calculated for all the bars used.

Improving the combination (3)

The object of this is to change the momentary combinations in their composition, in such a way that the deviations in their area are decreasing in magnitude. This is done as follows:

From the group of bars whose deviation has the largest absolute value, which may be referred to as the exchange group, the bar occupying a particular layer is tried in the corresponding layer in other groups (Fig. 3b). For each such experiment new deviations are obtained for both groups involved in the exchange, if the cavity areas of the interchanged bars differ from one another. Only those experiments are kept on record in which the absolute value of the new deviations are smaller than that of the extreme exchange group before the experiment. From the remaining opportunities for exchange each time, the one giving the best quality will be selected. This process is repeated for each layer until no further satisfactory exchange can be made.

Allocation of groups of bars to rotor slots (4)

All that remains now is to determine where the groups of bars visualized as slot fillings shall be

[1] The bracketed numbers refer to the similarly numbered boxes in Fig. 2.

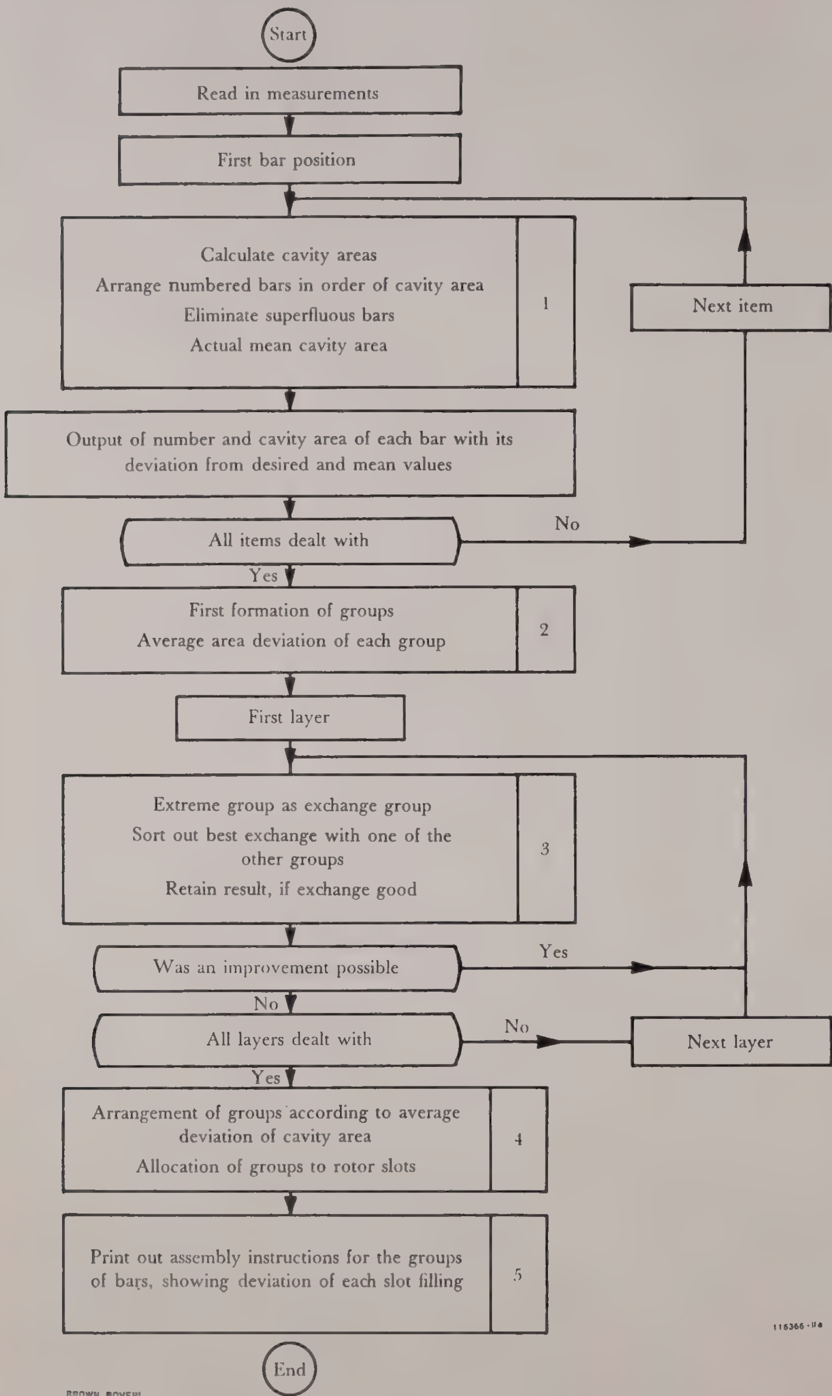


Fig. 2. – Block diagram of the computer program

incorporated. The object of guided insertion of the bar groups in the slots (Fig. 3c) is to eliminate, as far as possible, the effect of the small deviations between the groups which still remain in spite of the improvement process. This is achieved by placing groups of bars having almost equal quality factors in diametrically opposite slots.

Issuing the assembly instructions (5)

The computer prints out in clear the calculated composition of the slot fillings, their respective deviations and the allocated slot number. The resultant chart is then sent direct to the workshop as assembly instructions, according to which the assembly of the rotor winding can proceed.

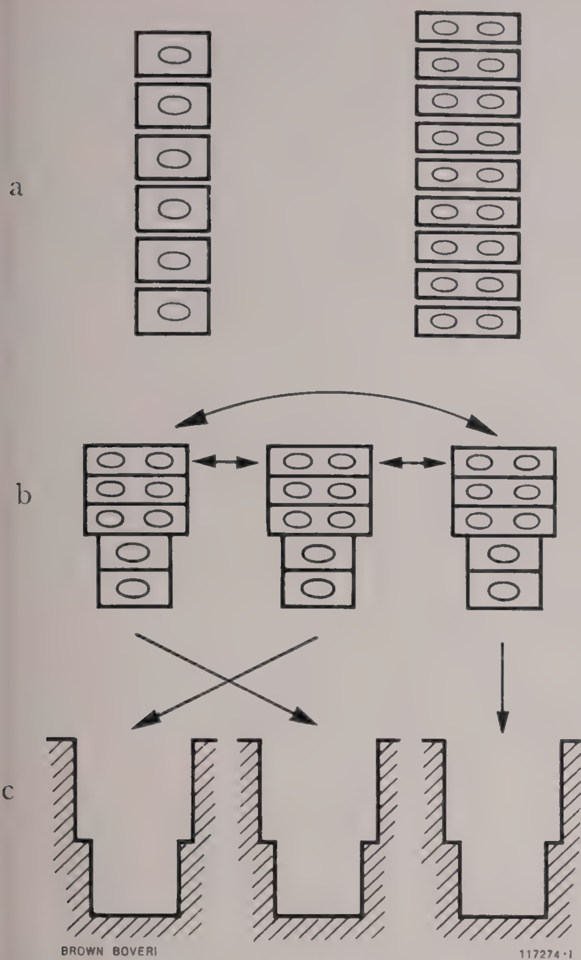


Fig. 3. – Diagram illustrating the principle of aimed bar distribution

a: Individual bars arranged according to item number and deviation
b: Groups of bars and lines of exchange during the improvement process in one layer
c: Allocation of groups of bars to rotor slots
The number of bars per slot and the number of bar items have been reduced for the illustration.

Effectiveness of the Program

The efficiency of the combination process described in the foregoing can easily be judged by comparing the stages before the formation of the groups and after they have been sorted out. Fig. 4 shows two distribution curves from a case which actually occurred in practice, from which the initial state and the effect of planned bar combination can be compared. Curve *a* shows the distribution of the individual bars as a function of their deviations, as measured. From a total of 576 bars it is required to form 64 groups. The r.m.s. error of the groups formed in the first step (Fig. 2, box 2) was about one-fifth of that of the individual bars. During the subsequent process of improvement (Fig. 2, box 3) the mean error in the cavity area was again reduced by 96 %. The distribution curve of the improved groups is shown as curve *b* in Fig. 4. Its mean error is extremely small. The digital computer goes one step further by arranging the groups of bars in the rotor slots in such a manner that the slot fillings cannot exert any further thermal disturbance effect.

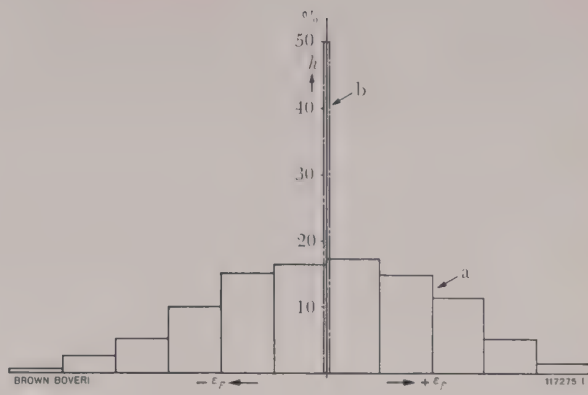


Fig. 4. – Distribution curves from an example in practice

h = Relative frequency
ε_F = Relative deviation of cavity area
Curve a: Distribution curve of the individual conductors
Curve b: Distribution curve of the slot fillings

A feature of the computer program described is that it contains rather more logical operations that arithmetical. The program comprises roughly 2600 instructions and, when processing the maximum permitted amount of information, requires about 5600 storage cells. Depending on the number of bars and the quality of the conductors, the time taken to complete the calculation may be between 70 and 120 minutes, which includes the process of printing out the results.

A suitable organization has been devised for the employment of the program and for punching the various measurements, with the result that the winding shop can be given the mathematically computed assembly instructions at the latest one day after receipt of the measurements of the hollow conductors. In this case the digital computer is rendering invaluable service in the course of a manufacturing process.

(KME) G. NEIDHÖFER

THE CALCULATION OF ELECTROMAGNETIC TRANSIENTS ON LINES BY MEANS OF A DIGITAL COMPUTER

681.14:621.315.064

The calculation of discontinuous transients on electric transmission lines and cables is very laborious in most cases. Following some introductory remarks on typical difficulties, the authors explain how the Schnyder-Bergeron method, originally devised as a graphical method, can be modified to suit the requirements of digital computers. Various examples which have been worked out with an appropriate program confirm the practical applicability of the method.

1. The High-Frequency Transient

IN ORDER that all machines and apparatus concerned with the transmission of electricity may be safely and rationally dimensioned, it is most important to know something about the overvoltages likely to be encountered. Herein a special role is played by the high-frequency overvoltages which occur after lightning strokes and certain switching operations. A characteristic feature of such phenomena is that the electrodynamic disturbance of the equilibrium of the lines appears in a wave form.

If full account is to be taken of this special character of the phenomena, the lines and cables must be regarded as a continuous structure with distributed constants. This applies when either analogue or digital computers are used. The strict mathematical treatment of continuous structures, such as lines, is known to be rather difficult, and almost impossible on an analogue computer.

By substituting chains of quadripoles for the lines in the calculation it is possible to find a way round the difficulties which prevent them from being regarded as continuous structures. In particular, the calculation on an analogue basis with the aid of the network analyser becomes feasible. But this procedure suffers from the serious drawback that, when travelling waves with a very steep wave-front have to be investigated, there is a risk of the calculation or the analogue simulation of the conditions at the wave-front being wrongly reproduced.

The method described below is based on the exact equations of the line with no losses and in simple cases is ideal for programming for the digital computer.

2. Various Possible Mathematical Methods of Solution

The equation of the line with no losses is the simplest partial differential equation of the second order of the hyperbolic type, indeed it is the first partial differential equation ever used [1]. D'Alembert (1717-83) and Daniel Bernoulli (1700–82) found two quite different ways of solving this equation, which is simultaneously the equation of the vibrating string. Bernoulli's solutions are founded on the eigen function and are comparable with the Fourier series. In contrast D'Alembert's method utilizes the wave character of the solutions of the hyperbolic differential equations.

It is one of the peculiarities of the hyperbolic differential equations that they can assume discontinuous solutions corresponding physically to travelling waves with an infinitely steep wave-front. The expansion of solutions according to Fourier series is not very suitable for representation of discontinuous functions because the series do not converge easily, and at the points of discontinuity do not properly reproduce the solutions, despite the fact that these solutions can formally be regarded as exact (Gibb's phenomenon) [2]. Moreover, a similar phenomenon is observed when, for the calculation, the line is replaced by a chain of quadripoles.

An exact reproduction of travelling waves with discontinuous points cannot be attained, even with very close sub-division. D'Alembert's method avoids this difficulty and is used as the basis of the method adopted herein.

For calculations in technical hydraulics the method was first devised by Schnyder [3] and Bergeron [4], working quite independently, and was visualized as a graphical method for the calculation of transients in penstocks. Programs for computation by digital computers already exist for such applications [5].[1]

These methods, devised by engineers, adopt a rather odd system of mathematical expression, but it is quite clear and will therefore be retained in this article. From the mathematical aspect it may be regarded as an application of the theory of characteristics, and it is possible to derive the basic formulae used from Riemann's method of integration for hyperbolic differential equations.

3. Fundamentals of the Schnyder-Bergeron Method

3.1 The Line Equations and D'Alembert's Solution

The following notation is used:

t – time

x = co-ordinate on the line

i = current, positive in the direction of positive co-ordinates

u = voltage to earth

L = inductance per unit length

C = capacitance per unit length

$Z = \sqrt{\dfrac{L}{C}}$ = impedance of the line

$c = \dfrac{1}{\sqrt{LC}}$ = speed of propagation of the phenomena along the line

For the line with no losses the following equations may be written

$$-\frac{\partial u}{\partial x} = L\,\frac{\partial i}{\partial t}$$
$$-\frac{\partial i}{\partial x} = C\,\frac{\partial u}{\partial t} \qquad (1)$$

By eliminating one of the two dependent variables, we obtain:

$$\frac{\partial^2 i}{\partial x^2} - \frac{1}{c^2}\,\frac{\partial^2 i}{\partial t^2} = 0$$
$$\frac{\partial^2 u}{\partial x^2} - \frac{1}{c^2}\,\frac{\partial^2 u}{\partial t^2} = 0 \qquad (2)$$

$F(y)$ and $f(y)$ are any two functions of the variable y. The two functions

$$i\,(x, t) = F\,(x - ct) + f\,(x + ct) \qquad (3a)$$
$$u\,(x, t) = Z\,F\,(x - ct) - Z\,f\,(x + ct) \qquad (3b)$$

separately satisfy the appropriate equation from (2) and both satisfy the system (1); they yield the general solution to the problem. $F(x - ct)$ describes a wave advancing in the positive x direction while $f(x + ct)$ describes one travelling in the negative x direction[2] (D'Alembert's solution).

3.2 The Fundamentals of the Schnyder-Bergeron Theory

Equation (3a) is multiplied throughout by Z, and then successively added to and subtracted from (3b), giving:

$$u\,(x, t) + Zi\,(x, t) = 2Z\,F\,(x - ct) \qquad (4a)$$
$$u\,(x, t) - Zi\,(x, t) = -2Z\,f\,(x + ct) \qquad (4b)$$

It may be observed that, in equation (4a), when $x - ct$ is constant, the left-hand side becomes constant; the same applies to the left-hand side of (4b) when $x + ct$ is constant.[3]

For an observer moving in the positive direction with the velocity c, the expression $u(x, t) + Zi(x, t)$ accordingly appears constant. The same applies to the expression $u(x, t) - Zi(x, t)$ for an observer in the opposite direction.

In an i, u plane (Fig. 1) $u + Zi$ = constant corresponds to the straight line g_+ and $u - Zi$ = con-

[1] The method given in the quoted paper is, however, based on the theory put forward by Allievi, which also relies on progressing waves, but which is purely analytical.

[2] The theory of hyperbolic differential equations shows that F and f may also be discontinuous [6, 7].

[3] $x + ct$ = const. and $x - ct$ = const. are the characteristics of the differential equation [6, 7].

stant to the line g_- with the same slope in the opposite sense, the sign referring to the direction in which the observer is travelling.

On the line $n+1$ points are given, namely $P_0 \ldots P_j \ldots P_n$ having the co-ordinates $x_0 \ldots x_j \ldots x_n$ (Fig. 2). $x_0 = 0$ and $x_n = l$ (the length of the line) coincide with the two end points P_0 and P_n. It is also assumed that

$$x_0 < x_1 < \ldots < x_j < \ldots < x_n.$$

τ_j denotes the transit time for the wave over the distance $P_{j-1} P_j$.

$$\tau_j = (x_j - x_{j-1})/c \tag{5}$$

Let us now consider three successive points P_{j-1}, P_j and P_{j+1} on the line, assuming provisionally that $j \geqslant 1$ and $j \leqslant n-1$ (i.e. P_j is an internal point). In this case it is presumed that all τ_j are whole-number multiples of the basic time τ_0:

$$\tau_j = m_j \tau_0$$

In the remarks which follow, only whole-number multiples of τ_0 will be considered

$$t = m \tau_0$$

using m as a new time-scale.

From the point P_{j-1} at the instant $m - m_j$ an observer is supposed to travel towards P_j, and at the instant $m - m_{j+1}$ another, also in the direction of P_j from P_{j+1}. The two will arrive at P_j at the instant m. The straight line g_+ corresponds to the observer moving from P_{j-1} in the positive direction of x, whereas the line g_- corresponds to an observer travelling in the opposite direction. Since the two observers, on arriving in P_j, both experience the same current and voltage, the co-ordinates of the point of intersection of g_+ and g_- are i and u, the current and voltage at P_j at the instant m.

$\psi_j(m)$ will now be taken to symbolize the state of the line at the point P_j at the instant m. The symbol stands for both the numbers $i(x_j, m)$ and $u(x_j, m)$. According to the remarks above, the following principle applies:

When $\psi_{j-1}(m - m_j)$ and $\psi_{j+1}(m - m_{j+1})$ are known, $\psi_j(m)$ is determined by a simple algebraic operation, the procedure for which is as follows.

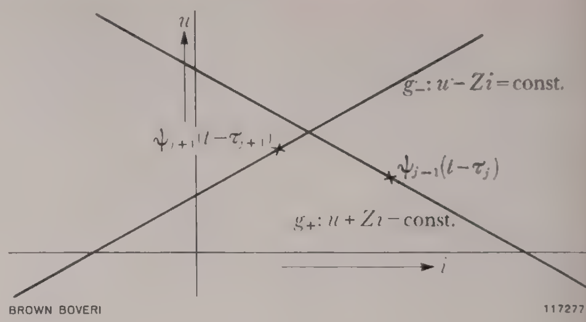


Fig. 1. – Explaining the Schnyder–Bergeron method

If an observer were to move along the line in the positive direction at the speed of propagation c, he would at all times experience the relationship between the voltage u and current i as expressed by the line g_+. Accordingly the line g_- would apply for a similar observer moving in the opposite direction.

Z = Line impedance

From $\psi_{j-1}(m - m_j)$ and $\psi_{j+1}(m - m_{j+1})$ we can determine the two straight lines g_+ and g_-:

$$\text{For } g_+ \quad a_+ i + b_+ u + c_+ = 0$$

$$\text{For } g_- \quad a_- i + b_- u + c_- = 0$$

The coefficients a_+, etc., are constants whose values can easily be calculated; they are governed by Z and the states $\psi_{j-1}(m - m_j)$ and $\psi_{j+1}(m - m_{j+1})$ (cf. Fig. 1). Resolved for i and u these equations provide the new state $\psi_j(m)$, which can be written symbolically as

$$\psi_j(m) = S(g_-, g_+)$$

In which S is a symbol for calculating the point of intersection of the two straight lines. The two ends P_0 and P_n must be dealt with separately because here boundary conditions must be taken into account.

In general form these are introduced as follows:

$$\varphi_j(i, u, m) = 0 \qquad j = 0, n$$

in which φ_j is a function of the three variables i, u and m.[4]

[4] In the largely automatic program described in section 4, the form $A_1(m)i + A_2(m)u + A_3(m) = 0$ is assumed for φ.

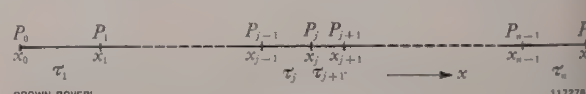


Fig. 2. – The line examined

P_j (where $j = 0 \ldots n$) denotes the point on the line having the co-ordinate x_j, and τ_j (where $j = 1 \ldots n$) the transit time of the wave between P_{j-1} and P_j.

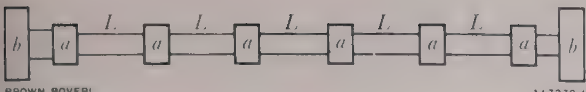


Fig. 3. – General arrangement of the line covered by the program

a = Quadripoles consisting of simple basic networks
L = Line or cable
b = Symbol for boundary conditions

These boundary conditions include such conditions as short circuit, line at no-load and connection of the line to an infinite bus at voltage u.

To determine the current and voltage at P_0, the following procedure is adopted.

For an observer moving from P_1 towards P_0 at the instant $m - m_1$ a straight line g_- applies. He arrives at P_0 at the time m, where the relationship $\varphi_0\ (i, u, m)$ applies. Hence the operation

$$S\ (\varphi_0, g_-)$$

yields the current and voltage at the time m. This time the S symbol is used in a somewhat extended sense. Analogue considerations apply to P_n.

3.3 Expansion of the Theory to General Arrangements

Fig. 3 shows the general diagram dealt with by the first program to be set up for problems of this kind. Line sections with generally different impedances are connected to one another by means of certain simple basic circuits. These circuits are referred to as quadripoles. In this general case too, the calculation of the conditions can be referred back to the S operation dealt with in the foregoing section.

The suffix j is now used to number the various junctions, each of which is allocated two states $\psi_{+,j}(m)$ and $\psi_{-,j}(m)$, as shown in Fig. 4. An ordinary point P_j on the line is dealt with as though the line sections to left and right of P_j were joined by an "empty" quadripole with direct-axis reactance = 0.

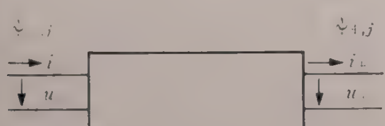


Fig. 4. – Notation of a quadripole

The functions $\psi_{-,j}$ and $\psi_{+,j}$ stand as symbols for the corresponding currents and voltages.

To calculate $\psi_{+,j}$ a modified straight line g_+^* is used which, together with g_-, determines the state $\psi_{+,j}\ (m)$ by means of the S operation. $\psi_{-,j}\ (m)$ can finally be obtained by simple algebraic operations.

If the quadripole consists of straightforward resistances, the coefficients of g_+^* are easily determined from g_+ and the coefficients of the quadripole, for which g_- represents an unmodified straight line used for the S operation at an ordinary point on the line. In this case the calculation is strictly correct.

If, on the other hand, the quadripole contains capacitances and inductances, the use of a modified straight line g_+^* is only correct as an approximation. In this case the coefficients of this line, in addition to the data of the quadripole and the coefficients of g_+, also contains the properties of state (currents, voltages) of the quadripole at the instant $t - \tau_0$, if the state at time t has to be calculated. Here τ_0 is the elementary transit time introduced in 3.2; thus all transit times τ_j are whole-number multiples of τ_0. The smaller τ_0, the better the accuracy of the approximation. The most suitable value for τ_0 will in most cases be determined by the quadripole with capacitances and inductances; regarding the difficult question of the correct choice of τ_0 some hints are given in section 5, with reference to examples.

3.4 The Block Diagram

Let us suppose m_{max} is the largest of the numbers m_j defined in 3.2. The switching action which initiates the transient takes place at $t = 0$. It is assumed that the states $\psi_{+,j}\ (m)$ and $\psi_{-,j}\ (m)$ are known for $m = -m_{max} \ldots -1$. The state of the line corresponding to the longest transit time $m_{max}\,\tau_0$ must therefore be known too. This is fulfilled when the transient follows a steady state. This assumption regarding the past history of the line may appear rather odd, since it ought to be sufficient to stipulate that the initial condition at $t = 0$ be known. However it must be borne in mind that, in accordance with the assumptions, only the states at the point P_j are known and not those of the whole line at the time $t = 0$; the states before that time provide the necessary additional information.

If the now known states in the past are imagined to be arranged in a rectangular matrix, in rows with respect to time in the correct chronological order,

the uppermost, empty line before the start of the calculation being allocated to $t = 0$ and the bottom line to $t = -\tau_0 m_{max}$. Beginning at one end of the line, the states at $t = 0$ following the switching operation can now be calculated, and the uppermost line thus filled out. Having displaced each row downwards by one position, as a result of which the bottom row can be omitted as superfluous, the top row can be calculated again in the same manner, thereby yielding the states at the time $t = 1 \cdot \tau_0$. Repetition of this process finally yields the states at any desired instant $t = m\,\tau_0$. Naturally in the computer the matrix corresponds to a definite storage sequence, for which additional storage cells have to be reserved for the properties of state of the quadripole with capacitances and inductances referred to in 3.3.

4. Short Description of the Program

The organization of the program is such that the cases occurring most frequently can be dealt with without additional programming being necessary, thus restricting the preparatory work to the production of a punched tape for the input data. Under certain circumstances it may be necessary to prepare special sub-routines for the boundary conditions.

The topological structure of the arrangements which can be handled corresponds to Fig. 3, in other words only lines with no branches can be dealt with. Up to 24 line sections with different impedances and different transit times bearing definite ratios to one another can be handled. Between each line section a four-wire junction is visualized. Not more than nine different types may occur, though the same type may of course repeatedly occur, with different constants. Combined on a sub-routine tape are the calculation instructions for the modification of the straight lines g, as mentioned in 3.3, for nine typical quadripoles. Each type is allotted a reference number from 1 to 9.

A punched tape is now produced, containing the following information: Transit times and impedances of the various lines, boundary and initial conditions, print-out characteristic. The quadripoles are defined by their data and computation instructions. The latter determine the sequence and types of the

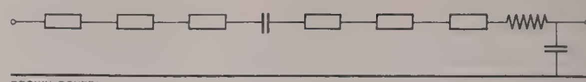


Fig. 5. – Representation of the examined line by a characteristic

The above diagram represents the characteristic 1 1 1 5 1 1 1 6.

various quadripoles; for example, 1 1 1 5 1 1 1 6 denotes an arrangement of the form illustrated in Fig. 5.

5. Examples

General Remarks regarding the Examples

The examples quoted were primarily used for checking the program and the method. In this case it is not enough merely to check the program for logical correctness. Most examples contain quadripoles for which approximation methods were used, as mentioned in 3.3. For such programs it is always essential to check through a few test cases, during which the necessary experience is gained, and confidence in the method.

Line Terminated by an Inductance (Fig. 6)

The line terminated by an inductance L and dead at $t < 0$, has a constant voltage $u = 1$ applied at $t = 0$. The current i through the inductance is to be calculated. This problem can be solved relatively easily with the method using eigen functions; the numerical solution is nevertheless so complicated that it is worth programming. The current through the inductance was deliberately calculated instead of the voltage, because the voltage, being a discontinuous function, as described in section 2, cannot be satisfactorily expanded with eigen functions.

The calculation was based on the following numerical values:

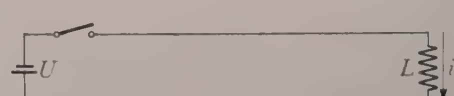


Fig. 6. – Check calculation to verify the program

At $t = 0$ the constant voltage U is applied to the line. The current in the inductance L is calculated.

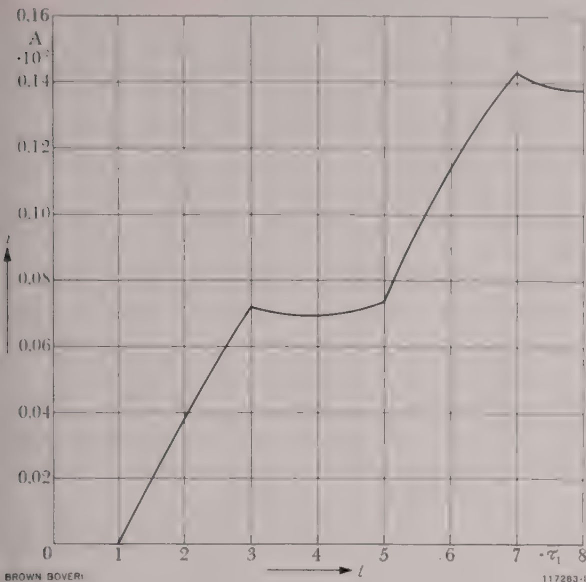


Fig. 7. – Result of the check calculation as in Fig. 6

Current i in function of time t. The time unit is τ_1, the transit time of the line. Calculation by the Schnyder-Bergeron method and the development of the solution using eigen functions gave approximately the same result.

Line length 1 km	($\tau_1 = 3{\cdot}33 \times 10^{-6}$ s)
Impedance	500 ohms
Inductance L	0·0167 H
Applied voltage U	1 V

Fig. 7 shows the curve of the current in the reactor as a function of time, the transit time τ_1 being chosen as time-scale. Up to $3\tau_1$ the calculation can be checked direct, since on expiry of one transit period

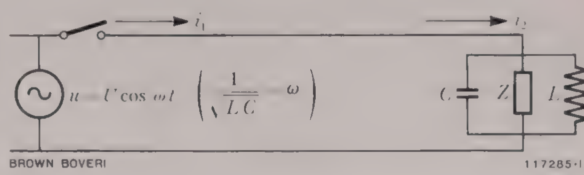


Fig. 9. – An example used to check the program

The line is terminated by a resonant network tuned to the power frequency. Parallel to the latter is a resistance, whose value is equal to that of the impedance. The line is connected to the a.c. source $\hat{U} \cos \omega t$. When the transient has decayed, the line transmits its natural power.

Length of line 750 km — Impedance $Z = 600$ ohms

τ_1, during which the current remains zero, it rises according to the well-known theory, in accordance with the law [8]

$$i = \frac{2u}{Z}\left(1 - e^{-\frac{Z}{L}t}\right)$$

This law remains valied until the wave, reflected at $t = \tau_1$ at the voltage source, returns for the first time with the opposite sign to the inductance L.

Since the time constant $\frac{L}{Z} = 3{\cdot}33 \times 10^{-5}$ s is large compared with τ_1, the current rises almost linearly. The rest of the curve can also be explained fairly easily.

Obviously the ratio of τ_0 to the time constant L/Z governs the accuracy of the approximation method described in 3.3. In the present case it was made very small, i.e. 0·002. Increasing it to 0·01 produced slight differences which are just visible in the trace.

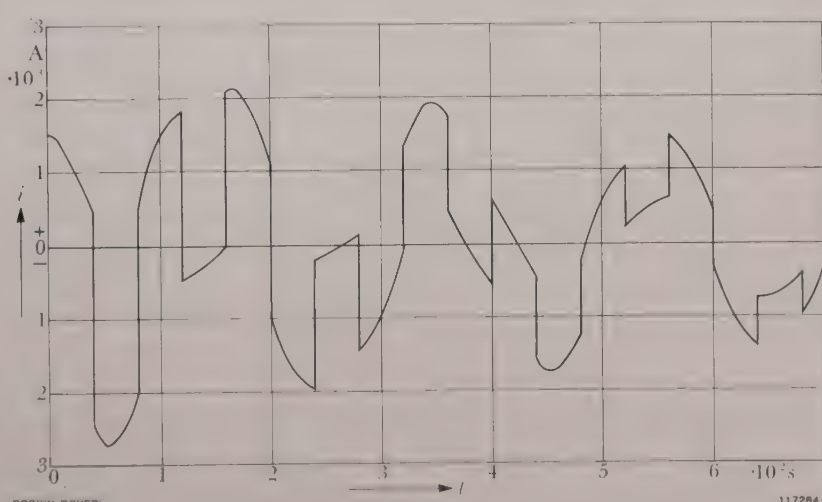


Fig. 8. – Approximate method of taking line losses into account

Showing the current at the infeed point when a 600 km line with losses is connected up. Due to the damping caused by the ohmic line losses, the discontinuity becomes gradually smaller.

$Z = 650$ ohms $\quad R = 0{\cdot}104$ ohm/km
$u = 1 \cos \omega t \quad \omega = 314\ \mathrm{s}^{-1}$

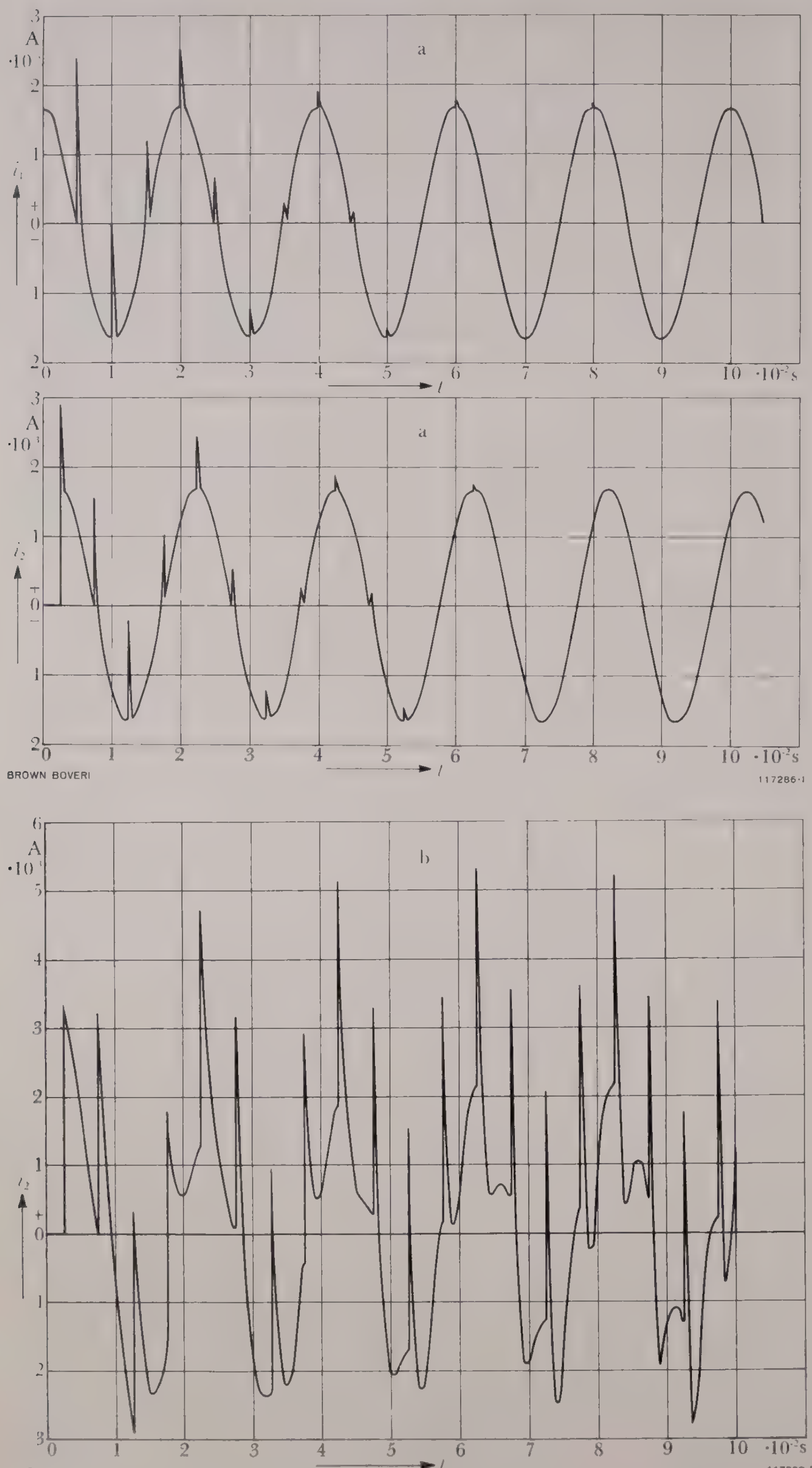


Fig. 10. – Curves showing the currents i_1 and i_2 in the circuit in Fig. 9, as a function of the time t

In Fig. 10a it can be clearly seen that the current, which is severely distorted to begin with, gradually reverts to the pure sine wave. (Small capacitance $C = 0{\cdot}203 \times 10^{-6}$ F.) In Fig. 10b hardly any damping can be detected. (Large capacitance $C = 0{\cdot}203 \times 10^{-4}$ F.)

Line with Losses

Line losses can be taken into account by dividing the line and connecting a resistance as a quadripole between each two sections. Calculations carried out on a 600 km long line proved that division into only 5 sections yielded almost the same results as a finer division into 15 sections.

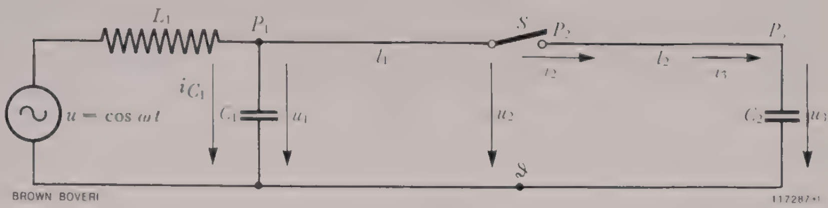


Fig. 11. – Investigation of transients following restriking in the breaker S

Before the restrike the two line sections $\overline{P_1P_2}$ and P_2P_3 are equally and oppositely charged. For notation, see text.

Fig. 8 shows the current produced when a 600 km length of line at no-load is connected to a 50-c/s supply at the instant when the voltage is at a maximum. It will be observed that discontinuity decreases with time, the curve approaching gradually to a sine curve.

Application of an A.C. Voltage to a Line Terminated with a Resonant Network

The arrangement is illustrated in Fig. 9. The resonance frequency of the termination is equal to the applied frequency of 50 c/s and the resistance is equal to the impedance. This theoretical example was employed for check purposes because the oscillatory state is very simple. As can be easily seen, it corresponds to the transmission of the natural power. The transient is damped by the resistance in the terminating network, in this case the impedance. During the calculation it was confirmed that the transients only decay in a short time, due to the damping effect, when the capacitance is sufficiently small. The damping effect of large capacitors is very slight because they absorb the incoming wave and very little energy is dissipated in the parallel resistance. Fig. 10a shows the currents at the ends of the line when the capacitance is relatively small. It can be clearly seen that the very discontinuous curve at the beginning is damped within a few cycles to such an extent that only the steady-state curve remains.

The step length τ_0 was made 10^{-4} s. In this case the time constant, formed by the impedance Z and capacitance C, governed the estimation of τ_0. (The time constant of the inductance is much larger in this case.) Since $C = 0{\cdot}203 \times 10^{-6}$ F, and $Z = 600$ ohms, $T_C = 1{\cdot}218 \times 10^{-4}$ s. Therefore τ_0 and T_C are both of the same order of magnitude.

By way of comparison, Fig. 10b shows the current i_2 at the line termination when the capacitance C is made 100 times larger. Here, for the reasons given above, the effect of the damping is almost imperceptible.

The Effect of Restriking in a Circuit-Breaker

According to Fig. 11 it is assumed that at $t = 0$ the breaker S is closed, the line sections on either side of the breaker being at equal and opposite potentials before closure.

Data: Impedance $Z = 500$ ohms

Line length $l_1 = 1$ km

$l_2 = 1$ km

Capacitance $C_1 = 800$ pF $= 8 \times 10^{-10}$ F

$C_2 = 1{\cdot}42$ μF $= 1{\cdot}42 \times 10^{-6}$ F

Inductance $L_1 = 0{\cdot}36$ H

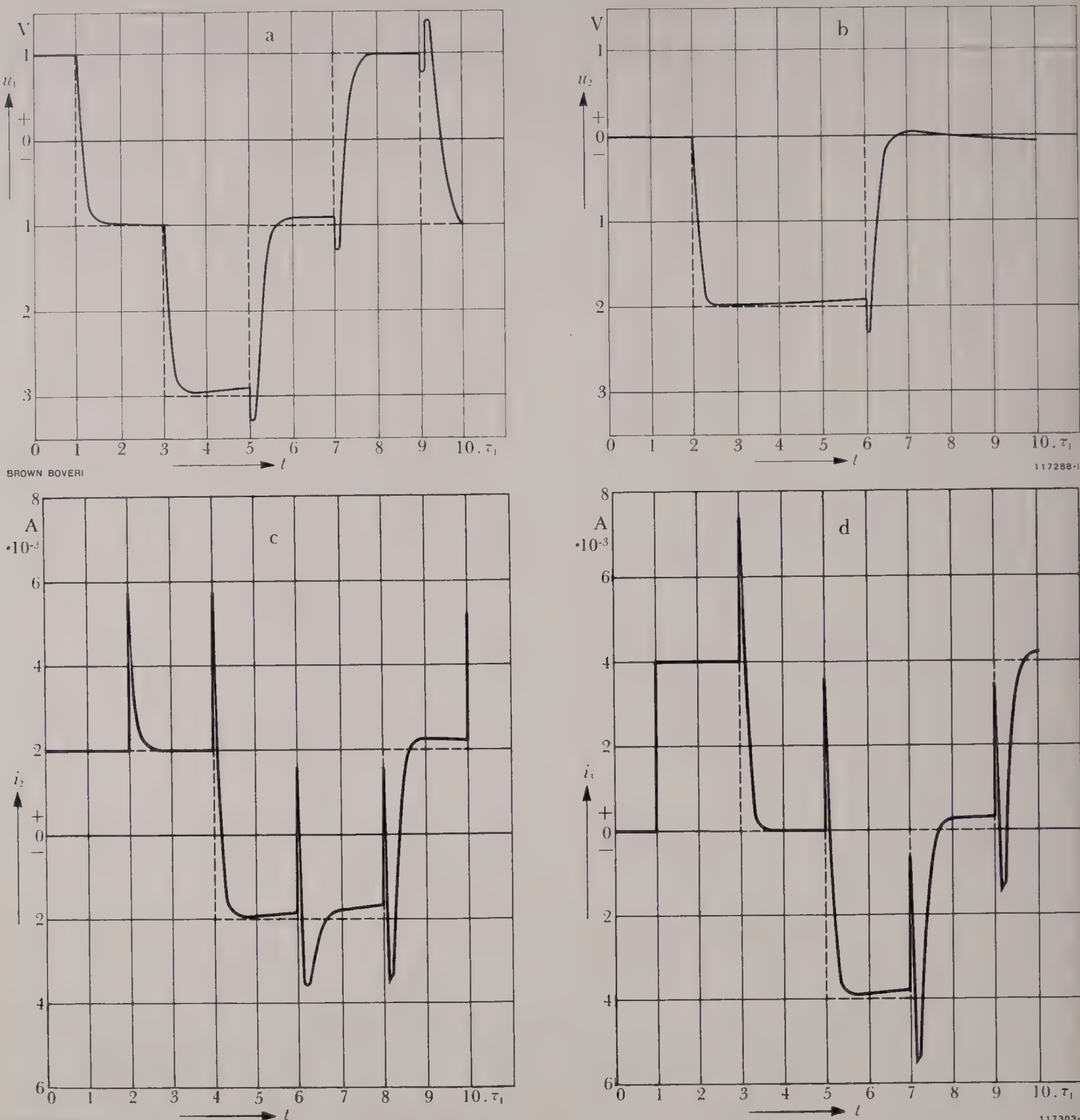
Voltages before closure of the breaker: [5]

Section $\overline{P_1 P_2}$ $+1$ V

$P_2 P_3$ -1 V

Fig. 12a to e show the curves of the current and voltage at the various points in the diagram. Again it can be established that, agreeing with the theory, the incidence of a wave at the capacitors results in an exponential shape of the current and voltage curves. Moreover, at P_1 the inductance is so high that, to begin with, it cannot affect this shape. The curves shown dotted in Fig. 12a–d correspond to the case in which the line is open at P_1 (small capacitance and high inductance) and short-circuited at P_3 (large capacitance). This case, which provides a very clear picture, makes it much easier to understand the complicated phenomena. For this example the choice of the basic transit time had to take into account the time constant $T_1 = C_1 Z$, which is very small. The ratio $\tau_0 : T_1 = 0{\cdot}139$ gave satisfactory results.

[5] Since the circuit is linear, the voltage level of the initial conditions can be selected arbitrarily.



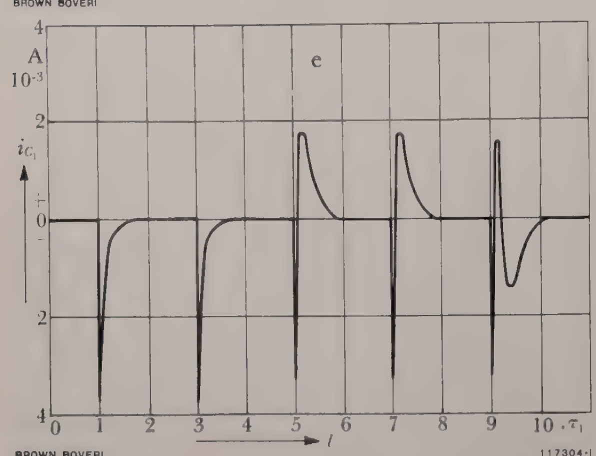


Fig. 12. – Transients caused by restriking

a: u_1 = Voltage at P_1
b: u_2 = Voltage at P_2
c: i_2 = Current at P_2
d: i_3 = Current at P_3
e: i_{C1} = Current in C_1

The time is shown as multiples of $\tau_1 = \tau_2$, the transit times of the two lines. The curves shown dotted indicate what happens when the line is open at P_1 and short-circuited at P_3.

6. Extension of the Program

Since the present article was first drafted, the program described in section 4 has been considerably extended. The conditions stipulated regarding the topology of the network, as shown in Fig. 3, were discarded. Now it is possible to investigate systems with any number of branches, as illustrated by a simple example in Fig. 13.

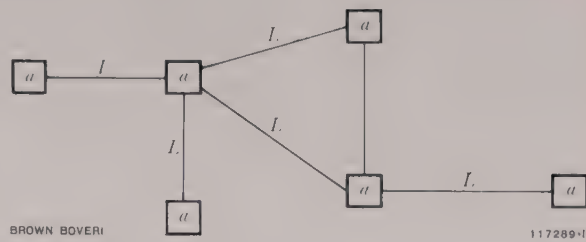


Fig. 13. – Example showing the topological structure of the system which can be examined with the aid of the extended program

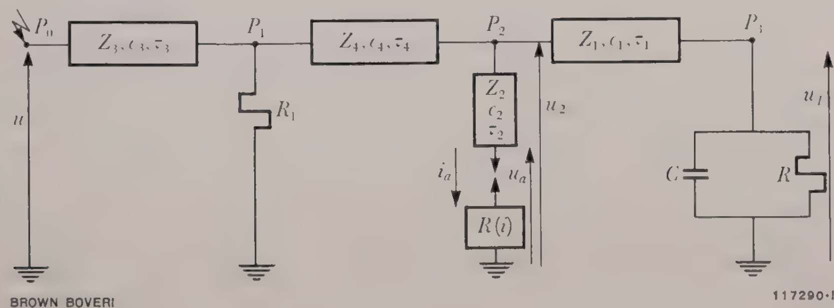
L = Line or cable

a = Electrical circuits or boundary conditions

The original intention was to utilize the program for the investigation of overvoltage phenomena in connection with line protection by lightning arresters. On this account it was necessary to make a further sub-routine capable of reproducing the properties of an arrester with reasonable accuracy. It is well known that an arrester acts as an insulator until the moment when the breakdown voltage is reached; subsequently the voltage is determined by the current flowing through the arrester. The discharge characteristic is represented in the program by a succession of straight lines, as shown in Fig. 15b, for instance.

The principle of the extended program will now be explained by reference to an example. The layout is illustrated in Fig. 14. At point P_3 there is a 220-kV transformer, represented by an input capacitance C and impedance R. At a distance of 15 m from the transformer, at point P_2, there is a branch leading to the lightning arrester 15 m away. 90 m before this branch, at P_1, are the connections to very long transmission lines. These can be represented by a resistance R_1 corresponding to their impedance. The cases investigated were $R_1 = \infty$ (no branches) and $R_1 = 200$ ohms (two parallel lines). At a point 600 m from P_1 lightning is assumed to strike the line at P_0, the amplitude being limited by the insulation level of the line. This was assumed to have the very high value of 1500 kV and the rate of rise of the incident wave-front 1000 kV/μs, a value commonly used for calculations when unfavourable conditions are anticipated. The shape of the incident wave is illustrated in Fig. 15a, while 15b shows the characteristic of the arrester. It was assumed that

Fig. 14. – Diagram for the investigation of overvoltage phenomena in a 220-kV installation



Point P_3: Connection to a transformer represented by C and R

Point P_2: Branch to a lightning arrester with non-linear resistance $R(i)$

Point P_1: Connection to transmission lines represented by the resistance R_1

Point P_0: Point treated as the start of the line in the event of a lightning stroke

Two examples a and b were calculated, using the following numerical values:

a: $Z_1 = Z_2 = Z_3 = Z_4 = 400$ ohms
$c_1 = c_2 = c_3 = c_4 = 300$ m/μs
$\tau_1 = \tau_2 = 0{\cdot}05$ μs $\tau_3 = 2$ μs
$\tau_4 = 0{\cdot}3$ μs; $R = 7$ kohm
$C = 1000$ pF
$R_1 = \infty$

For the fundamental shape of the voltage $u(t)$ see Fig. 15a; for the assumed arrester characteristic $u(i)$ see Fig. 15b.

b: As for a, but with $R_1 = 200$ ohms

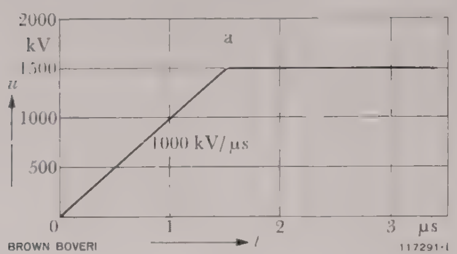


Fig. 15a. – Assumed shape of the voltage u(t)

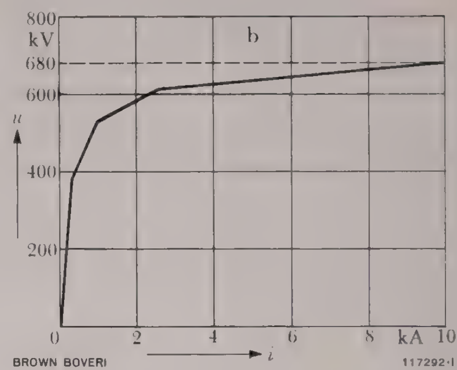
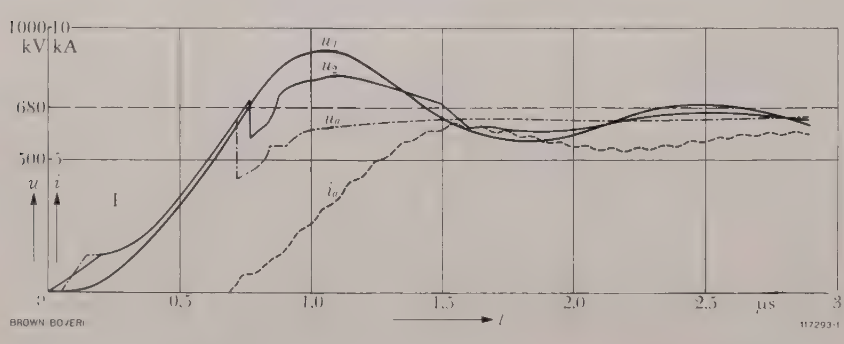


Fig. 15b. – Assumed characteristic u(i) of the arrester

the latter has a protective rating of 680 kV, conforming to the Swiss co-ordination regulations for 220-kV installations with effective earthing. Thus at the rated discharge capacity of the arrester, 10 kA, the residual voltage should just amount to 680 kV; the actual breakdown voltage is 660 kV. The quantities calculated were the voltages at the transformer (u_T) and arrester (u_a) and the current through the arrester (i_a). The results are plotted in Fig. 16, I and II. It will be observed in I that the voltage at the arrester never exceeds the protective rating. Since the transformer is 30 m away from the arrester, the voltage u_T rises briefly to 910 kV. According to a commonly used rule-of-thumb, for a distance of 30 m and a rate of rise of 1000 kV/µs, a brief voltage rise of $2\,\frac{l}{c}\,1000 = 2\,\frac{30}{300}\,1000 = 200$ kV may be expected. The calculation, however, proves that this rise may actually be somewhat higher. The illustrated voltage curve at the transformer corresponds to a full wave with an amplitude of about 800 kV. From Fig. 16 II it is possible to detect the influence of the parallel lines connected at P_1. The rate of rise of the incident wave is reduced by half. The result of this is that the maximum voltage experienced by the transformer is only about 785 kV. One might perhaps tend to expect a still greater influence from the parallel lines. The reason why this is not so is that the amplitude of the incident wave was assumed to be roughly twice as high as the protective rating of the arrester; hence two parallel lines are unable to reduce the voltages to such an extent that the arrester does not have to function.



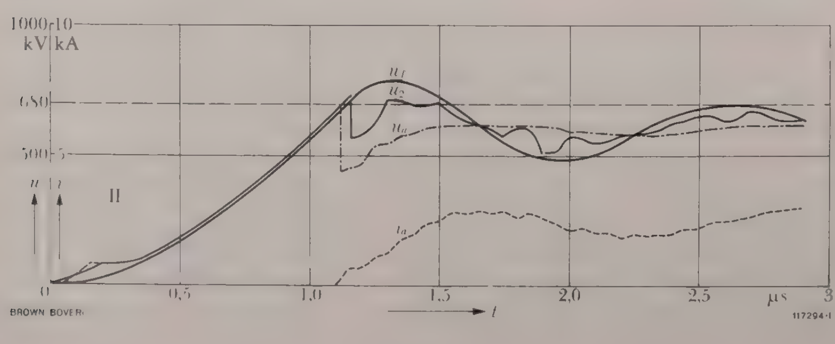


Fig. 16. – Calculated voltage/time curves at the transformer (u_T), at point P_2 (u_2) and at the arrester (u_a), as well as the current i_a through the arrester

I: for example a in Fig. 14
II: for example b in Fig. 14

7. Conclusions

Investigations carried out have confirmed that for discontinuous phenomena the method employed here is more suitable for the particular problems dealt with than the expansion of eigen functions. If the circuits contain capacitance or inductance, the Schnyder–Bergeron method can only be employed in practice on a digital computer because, in view of the accuracy required, the calculation has to be split into a very large number of steps. For this reason the time taken to complete some of the calculations listed in the article as examples was up to one hour. But it has been shown that a method originally intended as a graphical solution can be successfully adapted to render it applicable to a digital computer.

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(KME) P. Althammer

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INVESTIGATING THE TRANSIENT STABILITY OF SYNCHRONOUS MACHINES WITH THE AID OF A DIGITAL COMPUTER

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The article describes a program for calculating the transient stability of a synchronous machine connected through a general quadripole to an infinite bus. To illustrate applications, examples are quoted of common methods of calculating the stability and of the stability with automatic rapid reclosure.

1. Treatment of Stability Problems

THE CALCULATION of the behaviour of synchronous machines in the face of disturbances can only be performed in a reasonable length of time with the aid of electronic computers. Only isolated cases, involving say two or three machines, can be tackled rationally without such modern aids. But even in these simple cases quite extensive simplifications have to be made.

To determine the transient stability of a system with several machines the a.c. network analyser described in the article beginning on page 313 has given good results for some time, but lately the digital computer has begun to offer serious competition. The investigations on the analyser are also subject to quite considerable simplifications. For example, owing to these simplifications, it is impossible to determine the effect of the control system, the damping winding and even of the synchronous reactances on the transient stability.

The accuracy attained with the network analyser is generally adequate when the stability of a whole network has to be investigated. In most cases no higher accuracy could be expected from refined methods, because when whole networks are being examined some of the essential data are not known to any great accuracy.

However, in many cases an improved method for calculation of the stability is desirable, particularly when the influence of certain parameters on the stability has to be determined. In such problems it is often only a single machine or power station which is connected to an infinite bus by means of a transmission line. In such cases an ideal means is the analogue computer [1]. In certain cases, however, setting up the problem and obtaining the measurements on the analogue computer prove rather tedious and long-winded. A case in point is the problem of automatic rapid reclosure, where three different states have to be considered: normal operation, short circuit, interruption. If the effect of different parameters has also to be studied—involving the performance of a large number of calculations each time—the inautomatic operation of the analogue computer proves quite a serious handicap.

For these reasons a program for investigation of the transient behaviour of a single synchronous machine has been prepared. In contrast to the analogue computer, once a program has been checked, it does not occupy any further preparation time at the digital computer, from which arises the important advantage that relevant enquiries can be answered in quite a short time.

The mathematical fundamentals of the method and program will now be described, reference being made to the literature for certain details.

2. Stipulations for the Calculation

The arrangement handled by the program is illustrated in Fig. 1: A synchronous machine S is connected with an infinite bus N through a passive network represented by $V^{(i)}$. The voltage regulator and the exciter are regarded as being combined in the excitation circuit E. The superscript i in the

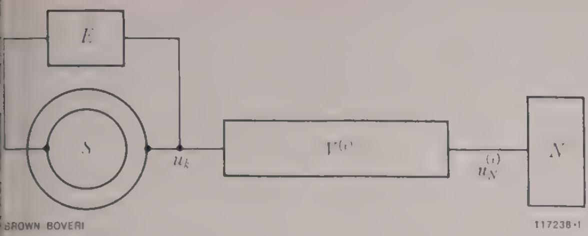


Fig. 1. – Schematic diagram to illustrate the set-up dealt with by the program

S = Synchronous machine

E = Excitation circuit, comprising exciter and regulator

$V^{(i)}$ = Quadripole representing the connection between the machine and the network with an infinite bus

N = Infinite bus

u_k = Terminal voltage

$u_N^{(i)}$ = Voltage of the infinite bus during the i-th state

i = Superscript denoting the system state

The quadripole $V^{(i)}$ may, for instance, represent a transformer with one or more lines. A fault in the system, e.g. interruption of a line, can be represented by an appropriate change in the quadripole. Loads can likewise be represented by the quadripole.

case of $V^{(i)}$ and $u_N^{(i)}$ indicates that, during the transient phenomenon, both the quadripole and the system voltage may vary. Henceforth the various states are referred to as system states, the state immediately preceding the disturbance being denoted by the superscript 0. When auto-reclosure is successfully executed, three different system states are involved, namely $V^{(0)}$ before the disturbance, $V^{(1)}$ during the short circuit, $V^{(2)}$ during the process of interruption; following reclosure $V^{(0)}$ is once more established. In the event of a persistent fault the sequence is $V^{(0)} - V^{(1)} - V^{(2)} - V^{(1)} - V^{(2)}$.

2.1 The Synchronous Machine

The mathematical representation of the synchronous machine is in accordance with the well-known Park's theory, the equations of which can, however, be considerably simplified for this purpose. Here reference is expressly made to [3], in which the equations and notation are given. As will be known, this theory also takes into account the effect of the damping windings in the two axes, each of these windings being represented by a circuit of its own. The saturation of the machine, on the other hand, is not allowed for, as is also the case in the present program. However, the capacity of the digital computer employed is large enough to permit extension of the program to include saturation.

To characterize the synchronous machine the following constants were fed into the computer:

x_d, x_q = synchronous reactances

x'_d, x''_d = transient and subtransient reactances in the direct axis

x''_q = quadrature-axis subtransient reactance

x_s = leakage reactance of the stator winding

T'_d, T''_d = direct-axis transient and subtransient short-circuit time constants

T''_q = quadrature-axis subtransient short-circuit time constant

T_a = starting time constant of the machine set

The above are known as the operational constants: apart from x_d, x_q and T_a, these constants are derived quantities which do not appear in the fundamental equations of the machine. For the sake of brevity the constants appearing in the fundamental equations will be called basic constants. These are the various leakage coefficients, x_d, x_q, and the time constants of the different windings, when the other circuits coupled to the particular winding are open-circuited.[1]

As was demonstrated elsewhere [2], the basic constants can be determined back from the operational constants, though for this an insignificant supplementary hypothesis has to be made. The resistance in the rotor of the machine is also neglected. If necessary, however, it can be taken into account in the quadripoles $V^{(i)}$. The program begins with the computation of the basic constants.

2.2 The Excitation Circuit

The parts of the program concerning the excitation are incorporated in the main program as subroutines, enabling a variety of excitation systems to be tried out, if required. In the examples quoted in the present article a regulator with proportional-integral action (PI), whose measuring system has a

[1] This is not fulfilled with the operational time constants T'_d, etc.

time constant, and an unsaturated exciter were assumed. The deflection of the regulator is limited in both directions.

2.3 The Quadripole $V^{(i)}$

The quadripole $V^{(i)}$ can be characterized by the four components of its iterative matrix (see page 359) which are complex in the general case; their dependence on frequency is not taken into consideration. Accordingly, for each system state, four complex numbers have to be stored. In order to give the program the necessary flexibility, the voltage of the infinite bus is also stored along with the components; this voltage may also be subject to certain fluctuations.

2.4 The Mathematical Method

The complete synchronous machine with damping windings in both axes leads to a differential equation of the 5th order. An assumption made regarding the excitation circuit was that it could be described by a system not exceeding the 10th order. The overall system of not higher than the 15th order is integrated step by step using Runge-Kutta formulae of the 4th order.

3. The System of Equations

3.1 The System for the Synchronous Machine

To formulate the Park system of equations, prior knowledge of which is taken for granted, the following notation is introduced:

Winding or axis:	Subscript[2]	
d axis of the stator	1	Direct axis
Field winding	2	Direct axis
Damping winding in the direct axis	3	Direct axis
q axis of the stator	4	Quadrature axis
Damping winding in the quadrature axis	5	Quadrature axis

i_j, where	$j=1\ldots5$	Currents
u_j, ,,	$j=1\ldots5$	Voltages ($u_3 = u_5 = 0$)
ψ_j, ,,	$j=1\ldots5$	Flux linkages
T_j, where	$j=2, 3, 5$	Time constants in the rotor windings
T_a		Starting time constant of the machine
m_A		Driving torque
ω_n		Rated angular frequency
δ		Angle between the induced voltage and the fixed system voltage
u_N		System voltage[3]
u_k		Terminal voltage
l_{jk}, where	$j, k=1\ldots5$	Induction coefficients

The induction coefficients are zero when j and k refer to two different axes, i.e. when $j \leqslant 3$ and $k \geqslant 4$, and vice versa. Apart from the time constants, all values are expressed in the per unit system (for details see [3]). The coefficients l_{jk} are either basic constants or simple combinations of these.

Employing this notation the system of equations for the machine can be written as follows:

$$\left.\begin{aligned} u_1 &= -\frac{1}{\omega_n}\cdot\frac{\mathrm{d}\psi_1}{\mathrm{d}t} + \frac{\psi_4}{\omega_n}\left(\omega_n + \frac{\mathrm{d}\delta}{\mathrm{d}t}\right) \\ u_4 &= -\frac{1}{\omega_n}\cdot\frac{\mathrm{d}\psi_4}{\mathrm{d}t} - \frac{\psi_1}{\omega_n}\left(\omega_n + \frac{\mathrm{d}\delta}{\mathrm{d}t}\right) \end{aligned}\right\} \quad (1)$$

$$u_j = i_j + T_j\,\frac{\mathrm{d}\psi_j}{\mathrm{d}t}\,,\; j = 2, 3, 5 \quad (2)$$

$$\psi_j = \sum_{k=1}^{5} l_{jk}\, i_k\,, \qquad j = 1\ldots5 \quad (3)$$

$$m_A = \psi_4\, i_1 - \psi_1\, i_4 + \frac{T_a}{\omega_n}\cdot\frac{\mathrm{d}^2\delta}{\mathrm{d}t^2} \quad (4)$$

Remarks regarding the system (1) to (4)

— In contrast to reference [3] in the bibliography, the time t is not introduced in the per unit system, but is expressed in seconds. For this reason ω_n appears sometimes in the denominator.

— In [3] the equation (1) contains $\frac{\mathrm{d}\vartheta}{\mathrm{d}t}$ instead of $\omega_n + \frac{\mathrm{d}\delta}{\mathrm{d}t}$ and in (4) $\frac{\mathrm{d}^2\vartheta}{\mathrm{d}t^2}$ instead of $\frac{\mathrm{d}^2\delta}{\mathrm{d}t^2}$. ϑ was defined as the angle between the field

[2] To economize in the notation it is preferable to employ numbers instead of letters as subscripts.

[3] Vectors of current or voltage are denoted by an arrow above the symbol.

winding and the winding axis of one of the three phases of the stator winding, wherein, without imposing any restriction on the generality, a two-pole machine is assumed. Therefore

$$\vartheta = \omega_n t + \delta - \frac{\pi}{2},$$

from which

$$\frac{\mathrm{d}\vartheta}{\mathrm{d}t} = \omega_n + \frac{\mathrm{d}\delta}{\mathrm{d}t} \text{ and } \frac{\mathrm{d}^2\vartheta}{\mathrm{d}t^2} = \frac{\mathrm{d}^2\delta}{\mathrm{d}t^2}$$

— In (1) the terms $\frac{\mathrm{d}\psi_1}{\mathrm{d}t}$ and $\frac{\mathrm{d}\psi_4}{\mathrm{d}t}$ describe the transformatory effect on the stator windings of changes in current in the rotor. In stability investigations it is permissible to suppress these terms. Likewise the derivatives of δ with respect to time may be neglected compared with ω_n.

Thus (1) simplifies down to

$$\left.\begin{aligned} u_1 &= \psi_4 \\ u_4 &= -\psi_1 \end{aligned}\right\} \qquad (1\,\mathrm{a})$$

According to the vector diagram (Fig. 2)

$$\left.\begin{aligned} u_1 &= u_k \sin\delta_m \\ u_4 &= u_k \cos\delta_m \end{aligned}\right\} \qquad (5)$$

in which δ_m is the phase angle between the induced e.m.f. $\vec{e}$ and the terminal voltage. If the machine is connected direct to an infinite bus, i.e. when $\vec{u}_k$ is constant, the equations (1a) and (2) to (5) are sufficient for calculating the phenomena in the synchronous machine. In this case $\delta_m = \delta$. This assumes that the voltage u_2 applied to the field winding and the driving torque m_A are given. If $\vec{u}_k$ is not constant, (5) must be replaced by a system containing the properties of the quadripole $V^{(i)}$.

3.2 The Equations of the Quadripole $V^{(i)}$

Using the notation in Fig. 3 the following equations apply for the quadripole:

$$\vec{u}_k = \overline{A}_{11}\,\vec{u}_N + \overline{A}_{12}\,\vec{i}_N$$
$$\vec{i}_k = \overline{A}_{21}\,\vec{u}_N + \overline{A}_{22}\,\vec{i}_N$$

The bars denote that these quantities are of a complex nature.

Substituting $\overline{D}_{11} = \frac{1}{\overline{A}_{22}}$ and $\overline{D}_{12} = \frac{\overline{A}_{12}}{\overline{A}_{22}}$

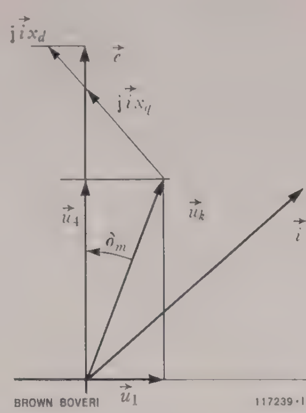


Fig. 2. – Vector diagram of the synchronous machine

In order to simplify the method of writing the equations, the voltages in the two axes are denoted by u_1 and u_4 instead of the usual u_d and u_q.

it follows that

$$\vec{u}_k = \overline{D}_{11}\,\vec{u}_N + \overline{D}_{12}\,\vec{i}_k$$

having regard to the fact that the determinant of the iterative matrix is unity.

This vectorial equation independent of the system of coordinates is resolved into two scalar equations. In the system of coordinates whose axes coincide with those of the machine, with the above notation, it follows that:

$$\overline{D}_{11} = \varrho + \mathrm{j}\xi, \quad D_{12} = r + \mathrm{j}x$$

$$\left.\begin{aligned} u_1 &= ri_1 - xi_4 + (\varrho\sin\delta - \xi\cos\delta)\,u_N \\ u_4 &= xi_1 + ri_4 + (\varrho\cos\delta + \xi\sin\delta)\,u_N \end{aligned}\right\} \qquad (6)$$

In the general case the equations (6) supersede those in (5).

3.3 Integration of the System

In the system (1a), (2), (3), (4), (6) only the equations (2) and (4) are differential equations. Let us substitute

$$v = \frac{1}{\omega_n} \cdot \frac{\mathrm{d}\delta}{\mathrm{d}t}$$

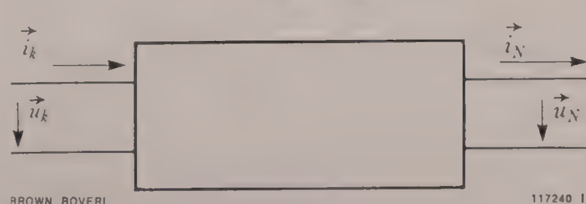


Fig. 3. Notation of the quadripole

in which v is the speed deviation during transient phenomena referred to the synchronous speed. Thus the differential equations of the system can be expressed in the form needed for integration

$$\left.\begin{aligned} \frac{d\psi_j}{dt} &= T_j^{-1}\,(u_j - i_j)\,, j = 2, 3, 5 \\ \frac{dv}{dt} &= T_a^{-1}\,(m_A - \psi_4\, i_1 + \psi_1\, i_4) \\ \frac{d\delta}{dt} &= \omega_n\, v \end{aligned}\right\} \quad (7)$$

The main variables

Of the various variables of the complete system, five, i.e. the three rotor fluxes, the angle δ and the speed deviation v, are specially distinguished, in that only derivatives of these variables occur. They will therefore be referred to henceforth as the main variables.

For the computation procedure it is now most important that, when the state of the system changes, these main variables do not change abruptly like the others. This can be seen from the above equations, which retain their validity when the state of the system suddenly changes. Since their right-hand sides remain finite, this excludes any abrupt change in the variables. For the quantities ψ_j this discovery complies with the principle of constant flux linkage. Since the derivatives of ψ_1 and ψ_4 are not taken into account in equation (1), the stator fluxes can, on the other hand, change abruptly.

The constancy of the main variables when the state of the system changes, i.e. corresponding to a change in the system of differential equations, renders it unnecessary to re-calculate the initial conditions after the change.

The main variables determine the state of the machine completely at all times. Only the main variables are stored in the program for the Runge-Kutta method.

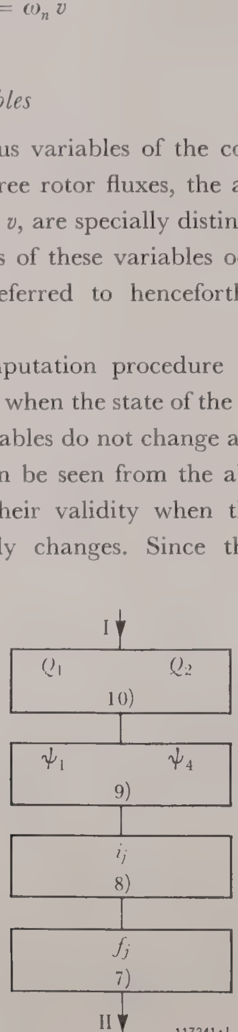


Fig. 4. – Flow diagram for calculation of the right-hand sides of the equations (7)

The quantities in the blocks are calculated in the indicated part of the program. The numerals refer to the relevant equations in the text.

From the organization aspect, this program is a sub-routine of the Runge-Kutta sub-routine.

I: from the Runge-Kutta sub-routine
II: return to Runge-Kutta sub-routine

Starting with these variables the right-hand sides of the system (7) are calculated at each step, according to the following procedure:

The system (3) can be resolved in terms of the currents i_j:

$$i_j = \sum_k \lambda_{jk}\, \psi_k \quad (8)$$

Since the matrix of the coefficients l_{jk} contains a large number of noughts, simple expressions can be used for λ_{jk} in the inverse matrix.

In (6) the substitution corresponding to (1 a) is made for the left-hand sides, while i_1 and i_4 are replaced on the right-hand side by the fluxes ψ_k, with the aid of equation (8).

We obtain a system of the form

$$\left.\begin{aligned} a_{11}\, \psi_1 + a_{14}\, \psi_4 &= Q_1 \\ a_{41}\, \psi_1 + a_{44}\, \psi_4 &= Q_4 \end{aligned}\right\} \quad (9)$$

with

$$\left.\begin{aligned} Q_1 &= a_{12}\, \psi_2 + a_{13}\, \psi_3 + a_{15}\, \psi_5 + \\ &+ \varrho\, u_N \sin\delta - \xi\, u_N \cos\delta \\ Q_4 &= a_{42}\, \psi_2 + a_{43}\, \psi_3 + a_{45}\, \psi_5 - \\ &- \varrho\, u_N \cos\delta - \xi\, u_N \sin\delta \end{aligned}\right\} \quad (10)$$

The a_{jk} bear a simple relationship to λ_{jk}, as well as to r, x, ϱ and ξ.

Resolving the system (9) for ψ_1 and ψ_4 gives the two stator fluxes as a function of the main variables. Inserting these values in (8) yields the currents i_j, so that now all variables have been determined as a function of the main variables. With these the right-hand sides of the equations (7) can now be calculated.

Fig. 4 shows the flow diagram of the rather complicated computation of the right-hand side of (7), denoted in the diagram by f_j. The necessary constants, such as a_{jk}, etc., were calculated before computation started.

The general Runge-Kutta sub-routine used is based on the following assumptions:

The system to be integrated is of the form

$$\frac{\mathrm{d}\, x_j}{\mathrm{d}t} = f_j\,(x_1 \ldots x_n)\,, j = 1 \ldots n.$$

The following information must be inserted:

Number of equations	n
Length of step	h
Initial addresses of the two series of variables	x_j and f_j
Address of a sub-routine calculating f_j from x_j each time	

When run through once the Runge-Kutta sub-routine, starting from the values x_j at time t, calculates the x_j at time $t + h$, the sub-routine being run through four times to calculate f_j.

3.4 System of Equations of the Excitation Circuit

The terminal voltage u_k needed in the event of voltage regulation works out in its absolute value to

$$u_k = \sqrt{u_1^2 + u_4^2} = \sqrt{\psi_1^2 + \psi_4^2} \qquad \text{(see equation 1 a)}$$

The control system with PI action utilized in subsequent examples gives three additional differential equations, one each for the time-lags of the measuring system, the exciter and the regulator with PI action.

The equations are simple and do not need to be listed here. Programming the limits merely needs a little thought.

3.5 Initial Conditions

The initial values of the main variables and the additional variables are computed by the program before integration starts.

For this, with the other data the following values must be given, referring to the state before the disturbance (superscript 0):

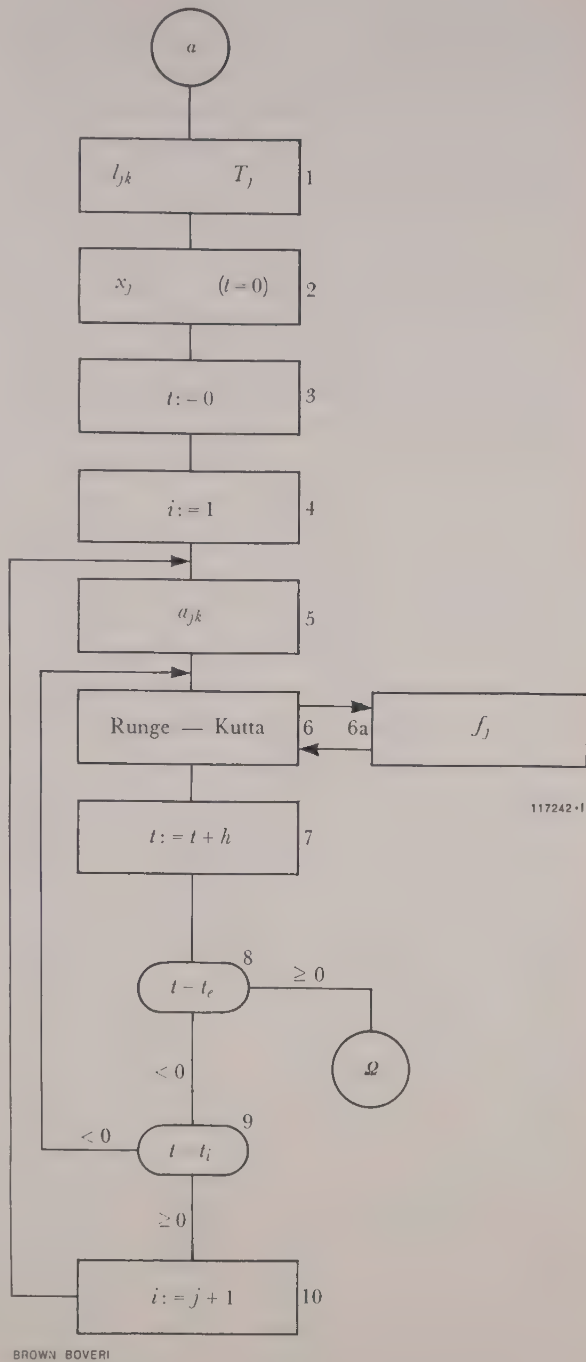


Fig. 5. – Flow diagram for calculation of the transient stability

Various details, such as printing instructions, have been omitted; likewise the links depending on whether a damping winding is provided or not.

p_0 = active power

$u_N^{(0)}$ = voltage of the infinite bus

$u_k^{(0)}$ = terminal voltage of the machine

From the machine data and the quadripole coefficients $A_{jk}^{(0)}$ of the quadripole $V^{(0)}$ we can calculate all the variables needed. Here it may be pointed out that, mathematically, the calculation yields two solutions; of these we have to choose the one corresponding to stable steady-state operation, because at least one of them is unstable in the steady state.

4. The Flow Diagram

Omitting details, Fig. 5 shows the flow diagram for the entire program.

4.1 General Remarks

The various states of the system are characterized by the subscript i, where $0 \leqslant i \leqslant 10$, to which a counting register is allotted. $i = 0$ corresponds to the state of the system before a fault or disturbance. t_i is the time at which the change from state i to state $i+1$ takes place, and is included with $A_{jk}^{(i)}$ in the input data; t_e is the time up to which integration is to take place.

4.2 Remarks on the Individual Blocks

Block 1: The basic constants are calculated from the operational constants (see section 2.1).

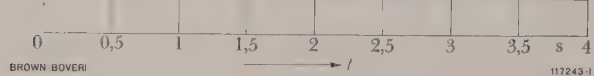


Fig. 6. – Short-circuit current during a three-phase short circuit when the synchronous machine is running at rated voltage

The curve is the envelope of the symmetrical component.

Machine data:
$x_d = 1{\cdot}6$ per unit
$x_d' = 0{\cdot}3$ p.u.
$x_d'' = 0{\cdot}2$ p.u.
$T_d' = 1{\cdot}0$ s
$T_d'' = 0{\cdot}02$ s

Before the short circuit the machine was running idle at rated voltage.

Block 2: The initial states are calculated from system state 0.

Block 4: The counting register is set to 1, thereby inserting the first state after the disturbance in the calculations.

Block 6: The Runge-Kutta sub-routine

Block 6a: Sub-routine according to Fig. 4.

The blocks 2 and 6a also contain sub-routines for the excitation circuit.

5. Examples

5.1 Determining the Symmetrical Component of the Short-Circuit Current in the Event of a Three-Phase Short Circuit

For $V^{(0)}$ the constants of a quadripole are inserted, given by a direct-axis reactance of $x_L = \infty$ (in practice it is 10^{10}). $V^{(1)}$ consists of a quadripole with the quadrature-axis reactance $x_Q = 0$ (in practice 10^{-10}). The sub-routine for the excitation circuit corresponds to a constant voltage applied to the field of the synchronous machine, made equal to 1 per unit. With these assumptions the computer calculates the envelope of the symmetrical short-circuit current. The asymmetrical component is not taken into account because, as explained in 3.1, the transformer effect is neglected.

Fig. 6 shows the well-known curve of the short-circuit current. This curve can be easily checked from the operational coefficients.

Calculation on the digital computer gave the following relative errors compared with calculation according to the formulae:

Time s	Relative error
0·04	0·02049
1·00	$< 10^{-5}$
4·00	$< 10^{-5}$

Whereas for longer times the error is negligibly small, at 0·04 s after the short circuit there was an error of 2 %. The calculation was carried out with a step length of 0·02 s, i.e. equal to the subtransient time constant. The error determined for 0·04 s, as was also confirmed by later tests, is connected with the rapid change in the current in the damping winding.

[T]he obvious rule may be made that the length of [t]he step should not be shorter than the subtransient [t]ime constant. A remarkable fact is that an initial [e]rror is not propagated over long periods.

5.2 Examples showing the Procedure for Stability Calculations

As mentioned at the start, simplifications are [u]sually made when calculating the transient stability. [C]onsidering a concrete example an investigation [w]as undertaken to check the effect of this on the [c]urve of the rotor angle with respect to time.

It is assumed that a machine is connected to an [i]nfinite bus by a two-circuit line 200 km long. A [t]hree-phase short circuit is assumed to occur at the [m]iddle of one of the two lines. After 0·15 s the [a]ffected line is locked out. The curves of the rotor [a]ngle during this disturbance were calculated (Fig. 7). [F]or the four curves the following conditions apply:

[1]: The system of equations must be complete.

[2]: The system of equations must be complete, but $T''_d = T''_q = 0$, thereby eliminating the effect of the damping winding.

[3]: The calculation is performed with constant flux linkage, without the influence of the damping winding. To obtain this variant we merely have to substitute $T'_d = \infty$, $T''_d = T''_q = 0$.

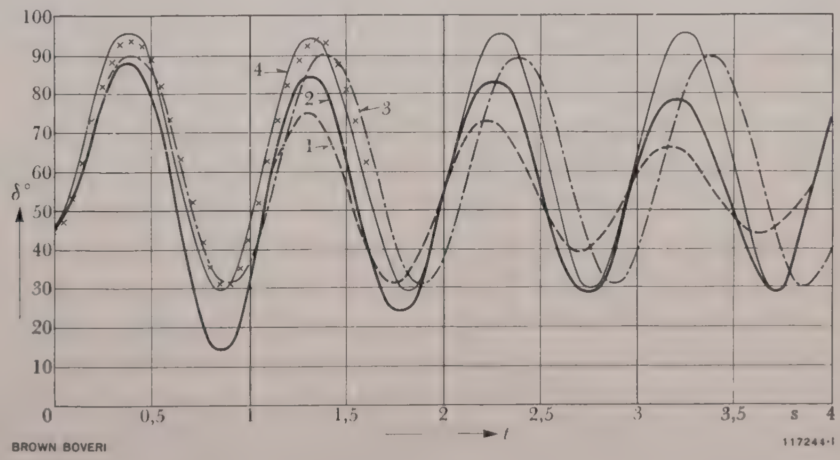
4: Calculation conforms to the simplification commonly used in practice, wherein only the transient reactance is effective in the two axes of the machine. The variant was calculated with the following stipulations:

$T''_d = T''_q = 0$, $T'_d = \infty$, $x_q = x'_d$, x_d may have any value but it must be $> x'_d$.

With this simplification the rotor angle is defined as the phase angle between the "air-gap voltage" following x'_d and the fixed system voltage. During hunting it differs from the true rotor angle by a constant amount. This difference was not taken into account in Fig. 7.

Data of the machine

Machine rating	= natural power capacity of the two-circuit line
Synchronous reactances	$x_d = 1{\cdot}0$, $x_q = 0{\cdot}65$ per unit
Transient reactance	$x'_d = 0{\cdot}3$ p.u.
Subtransient reactances	$x''_d = 0{\cdot}2$, $x''_q = 0{\cdot}2$ p.u.
Leakage reactance of stator	$x_s = 0{\cdot}17$ p.u.
Transient time constant	$T'_d = 1$ s
Subtransient time constants	$T''_d = 0{\cdot}02$ s, $T''_q = 0{\cdot}02$ s
Starting time constant	$T_a = 7$ s



[F]ig. 7. – *Rotor angle as a function of time*

[T]hree-phase short circuit on one of [t]wo parallel lines, cleared after [0]·15 s by disconnecting the affected line.

Length of line 200 km.

[C]alculation assuming certain conditions.

[1] = Complete system of equations
[2] = Neglecting the damping winding
3 = Constant flux linkage in the direct axis
4 = Calculation with x'_d in the direct and quadrature axis
× = Points calculated with slide-rule by the step-by-step method

Excitation circuit

Transfer function =

$$= K \cdot \frac{1}{1+pT_1} \cdot \frac{1}{1+pT_2} \left(1 + \frac{1}{pT_s}\right)$$

$K = 10$, $T_1 = 0{\cdot}1$ s, $T_2 = 0{\cdot}2$ s, $T_s = 2$ s

Maximum exciter voltage + 5 p.u.
Minimum ,, ,, − 1 p.u.

From curves 1 and 2 it will be observed that the damping winding only begins to take effect after one full oscillation, discernible in the increased damping of the oscillations.

Curve 3, which for short times coincides with 1 and 2, shows that, to begin with, the machine exactly conforms to the principle of constant flux linkage. Of course, in this case the oscillations are undamped. Whereas in 1 and 2 the voltage regulator in time corrects the terminal voltage back to the rated value, this is not the case in 3 because in this calculation the effect of the regulator was ignored. This explains why the mean value of the rotor angle is smaller in 1 and 2 than it is in 3.

Finally curve 4 shows the curve of the rotor angle, as obtained with the greatest simplification. It is surprising to note that the deflection of δ is larger than in curve 3. It can, however, be shown that immediately following the short circuit the power output, when calculated according to condition 3, is about 27 % higher than according to 4.

The points (x) plotted in curve 4 were obtained by calculation with a slide-rule, using the normal, simple step-by-step method (step length 0·05 s). It is evident that this method yields surprisingly good results within the scope of the conditions stipulated. For calculations on the network analyser the same procedure is adopted.

5.3 The Transient Stability Limit with Rapid Auto-reclosure

5.3.1 *Effect of the synchronous reactance on the transient stability in a special case*

The same conditions apply as in 5.2, i.e. the synchronous machine is connected to an infinite bus by a two-circuit line, the length of which is varied between 100 and 300 km, and a short circuit is assumed to occur at the middle of one line. But, in contrast to 5.2, it is assumed that after the line ha been interrupted, the short circuit is cleared and th affected line can be reclosed after a dead time o 0·3 s. By using a program in which the origina program appears as a sub-routine it would b possible to work out the maximum power which ca be stably transmitted under the particular conditions Thereby the length of the line and the synchronou reactances x_d and $x_q = 0{\cdot}65\ x_d$, were varied. Th remaining constants are the same as in 5.2.

It must be pointed out that it does not entirel conform to the technical circumstances to vary th synchronous reactance while keeping the transien reactance x'_d fixed. However, in these calculation the main point was to determine the effect of th synchronous reactances on the transient stability For the same reason, powers were considered whic exceeded the rated output of the machine. Fro Fig. 8 it can be seen that the transmissible powe P_W drops by about 15 % when the synchronou reactances are increased by a factor of 3·3, fro 0·75 to 2·5 per unit. Although this dependenc is not very marked, this result deserves commentin upon since sometimes the view is expressed tha in principle, the synchronous reactance can ha no effect on the transient stability. If, as und condition 4 of section 5.2, we calculate with x in both axes, there can be no dependence on th synchronous reactance. As was proved with referenc to Fig. 7, however, the stipulation of constant flu linkage gives a much closer approximation to th true curve. With constant flux linkage the activ power output of the machine is an unambiguo function of the rotor angle δ. Fig. 9 shows this fun tion for a definite case in the transmission syste investigated in this chapter, where the line leng is 300 km and the transmitted power 1 per uni The curves show the actual output of the machi during the various system states in terms of δ. T curves were calculated first for $x_d = 0{\cdot}75$ and afte wards for $x_d = 2$.

A_1 and A_2 are working points of the machi before the short circuit occurs. The value 1 − before the short circuit occurs. The value 1 − is the power tending to accelerate the machine. T higher the curves are situated and, in particular, t larger the included area, the better the transie stability of the machine, as will easily be realize

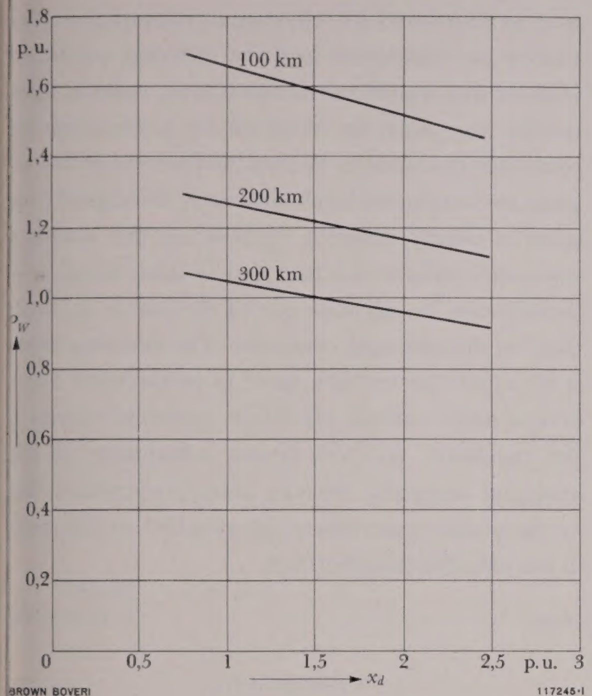


Fig. 8. – Curves showing the active power P_W which can be stably transmitted in the event of a three-phase short circuit with three-phase auto-reclosure, shown in terms of the synchronous reactance x_d for a two-circuit transmission system

Short circuit at the middle of the line. Line lengths 100, 200, 300 km.

Duration of fault = 0·15 s
Dead time = 0·3 s

The superiority of the machine with the smaller x_d (curves 1) stands out in Fig. 9. At this point it may be underlined that the corresponding curves for the calculation with x'_d in both axes of both machines would have the same shape.

5.3.2 The gain in stably transmissible power as a result of three-phase rapid auto-reclosure

Fig. 10 reproduces the results of calculations with the same arrangement as in Fig. 9. For a line length of 300 km the stability limit was computed once with and once without rapid auto-reclosure following interruption of the affected line, as well as with and without voltage regulation. A striking feature is that in this case the gain in stability resulting from auto-reclosure is relatively small. Here reference may be made to two publications in which analogous discoveries were made ([5], Fig. 2b on page 518; [6], page 152). It is not surprising that the voltage regulation has a certain influence on the stability, following the remarks in 5.2.

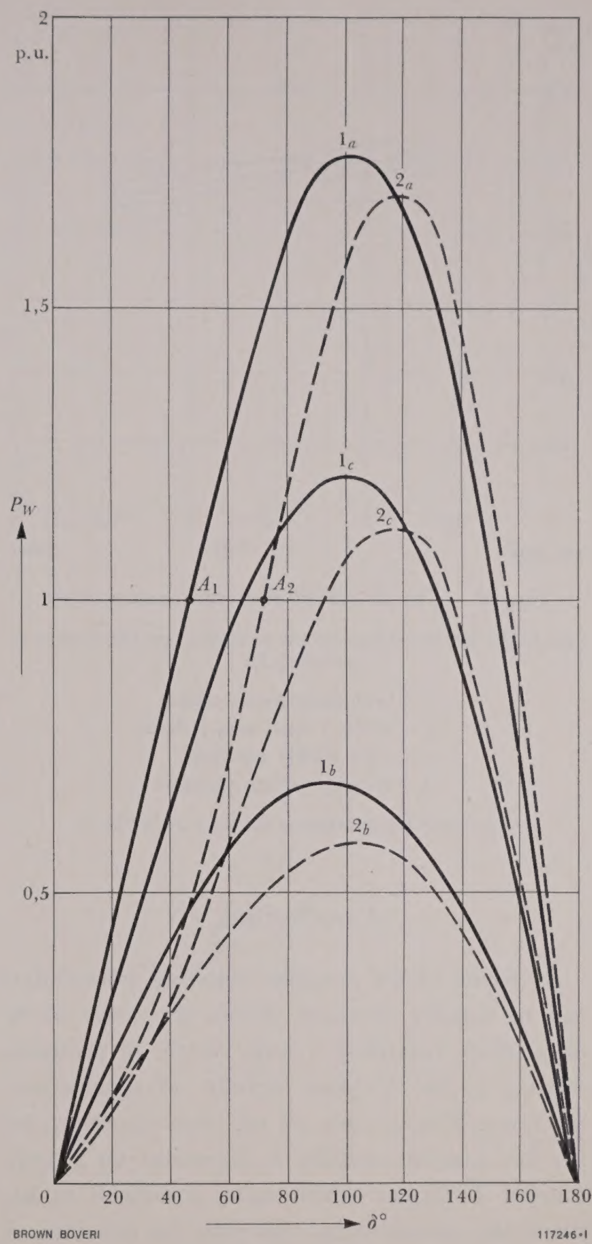


Fig. 9. – Explaining the dependence of the transient stability on the synchronous reactances x_d and x_q

Active power output P_W as a function of the rotor angle δ, assuming constant flux linkage. Line length 300 km.

$1 = x_d = 0{\cdot}75$ p.u. $\quad x_q = 0{\cdot}65 \times 0{\cdot}75$ p.u.
$2 = x_d = 2{\cdot}00$ p.u. $\quad x_q = 0{\cdot}65 \times 2{\cdot}00$ p.u.

a = Normal operation
b = Short circuit
c = One line disconnected

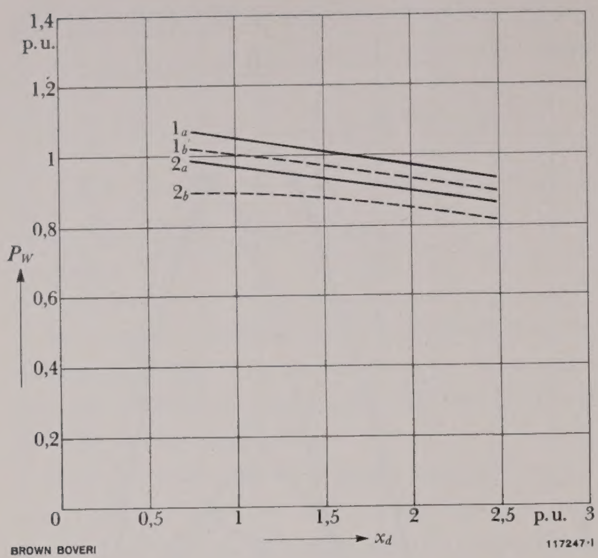


Fig. 10. – Gain in stability from rapid auto-reclosure

Line length 300 km, otherwise the conditions are the same as in section 5.3.1

1 = With rapid auto-reclosure
2 = Without rapid auto-reclosure
a = With voltage regulator
b = Without voltage regulator

The remaining notation is the same as in Fig. 8.

Conclusions

By means of the program described information can be quickly obtained, within the scope of its capabilities, regarding a wide variety of problems relating to the transient stability of synchronous machines. This is made all the more important by the fact that the stability is dependent on a large number of factors, which make it difficult to lay down any general rules regarding the influence of different parameters. The effect of auto-reclosure, the synchronous reactances and the voltage regulation, as discovered by reference to various examples, cannot be transferred to quite different conditions without due regard to the conditions; reliable information can only be obtained by performing the respective calculation. Within the limits to which the program is subjected, calculation by the digital computer is clearly superior to that on the analogue computer, despite the fact that it takes roughly 20 times longer to calculate one of the curves in Fig. 7 than on the analogue computer. The deciding factor is the much shorter time spent in preparation. However, if great value is placed on optimum setting of the regulator, the well-known advantages of the analogue computer become more pronounced. But no particular importance was attached to this point in the calculations described.

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